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STAR LOOK ANGLE COMPUTATION

Federal Electric Corporation
Vandenberg AFB, Calif. 93437

15 August 1977

UNLIMITED DISTRIBUTION

Prepared for

SPACE AND MISSILE TEST CENTER
Vandenberg AFB, Calif. 93437

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This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

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Star Look Angles	Proper Motion	Besselian Day Numbers
Astronomic Site Coordinates	Diurnal Aberration	Independent Day Numbers
Polar Motion	Mean Place	Computer Processing
UT1 Time	Observed Place	Encoder Bit Size
Parallax	Apparent Place	Octal Output

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The STAR 1program was prepared to replace a current SAMTEC Star Look Angle program run on the IBM 7094 computer by a program of higher accuracy which runs on the IBM 360/65 computer. The program is organized in module form for easy use in other programs. The design accuracy (neglecting refraction corrections) is 0.01 arcseconds for both azimuth and elevation angles. This program, with the proper BLOCK DATA, will produce look angles at a given site for a given time block (in hours) for as many days as desired, up to January 1 of the next year. → next page

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20. The coding was prepared in FORTRAN IV. Running speed is about 1.4 CPU seconds per star-station-day with polar motion corrections and a 1/2 hour integration step size.

ACKNOWLEDGMENTS

A special expression of appreciation is due Fred Grimes, whose time, effort, and contribution to this report could easily qualify him as a coauthor. The author would also like to thank John McNabb and Terry Bagley for their help in obtaining STASHO (WANG version) runs for use in comparisons with STAR1 runs. In particular, the author expresses his appreciation to John McNabb for the time spent with and documents given to the author in explanation of STASHO processing. The author would also like to thank Mike Walsh and Eugene Brick for their help in obtaining FPQ-14 Sigma 5 star position data for use in verification of STAR1. Special thanks is also due Mrs. Gwendolyn Jones for her many hours of painstaking effort for the preparation of this report.

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1.0 INTRODUCTION

Accurate pointing information to selected stars is required by various SAMTEC instrumentation for data acquisition and instrumentation calibration. Until now, the program STASHO on the 7094 computer was used for this purpose. While valuable when originally designed, new algorithms and programming techniques developed by the Naval Observatory have improved the accuracy obtainable by programs like STASHO. Operationally, the large amount of input information required for STASHO runs left a wide margin for the occurrence of human error.

For these reasons it was decided to design an observed position star look angle program based on the mathematics produced by the Naval Observatory (reference 1). This work was completed under the ROTI Capability Task (A7135) and under the Data Product Line Enhancement Task (A7102). It was decided to include precession-nutation and Vernal Equinox data internally within the program. An accuracy design goal of 0.01 or less arcseconds for the non-refracted pointing information was selected. Accordingly, all physical effects (except refraction) that can contribute more than 0.005 arcseconds are corrected. Since it is preferable to remove refraction errors from measured pointing angles by use of Rawinsonde data before regression, it was decided to use only a minimal refraction model in this program. However, since many models of refraction effects are available at WTR, this module was programmed so that additional refraction techniques can easily be added by the user as subroutines.

As work in the STAR program progressed, it became evident that additional programs were needed to supplement the original observed position star look angle program. The complete list of programs is:

- STAR1 - This program takes mean star coordinates referenced to the nearest Besselian New Year and computes azimuth and elevation data for a given site during a given time period.
- STAR2 - This program takes mean star coordinates for a given epoch and transforms them into mean star coordinates at a new epoch.
- STAR3 - This program computes 3/5 of the yearly data deck for STAR1 for any given year and outputs the data on punched cards.

JJJPP - This program takes a new BLOCK DATA (for use in STAR1) and produces tables to check against the tables available in the American Ephemeris and Nautical Almanac for the year in question.

Although the other programs have been checked out, this report will be restricted to the discussion and verification of STAR1.

The other programs were much easier to verify, in any case. Moreover, a listing of JJJPP can be found in Appendix VII. The operating instructions for STAR2 can be found in Appendix II. For those who wish the output of STAR3, please contact Dr. Sinclair at FEC/Performance Analysis Department. Also, look at Appendix V, which concerns use of the output of STAR3.

1.1 Outline of Report

Section 2 is devoted to a discussion of the processing and mathematics in the STAR1 MAIN program. The mathematics and processing involved in the various subroutines are outlined in Section 3. Numerical tests and numerical examples of the procedures unique to STAR1 as compared to STASHO (the present star look angle program used at WTR) are presented in Section 4. Also included in Section 4 is a comparison of the various types of Polar Motion data. The many subroutine outputs are verified in Section 5. Finally, the complete program verification and the report conclusions can be found in Section 6. The technical details of running the program (operating instructions, flow diagrams, listings, sample output, etc.) are relegated to the appendices.

2.0 MAIN PROGRAM DESCRIPTION AND MATHEMATICS

The STAR 1 program consists of a main program and nine subroutines. The routines and their functions are:

- . MAIN — Handles all I/O, initial and time dependent preprocessing, and the primary mathematics required for updating star positions.

- . ASDCRK — This subroutine transforms the Right Ascension, declination coordinate system to cartesian coordinates or vice versa.

- . CNSTNT — This subroutine updates the earth's obliquity, the equation of equinoxes and its first derivative, and the secular changes in the earth's rotation rate. It also computes the first and second differences for the updated Besselian constants A, B, C, D, and Independent constant f.

- . CNSTJ — This subroutine is an additional entry within CNSTNT. It computes second-order corrections on a ten-day basis.

- . OCT — This subroutine converts degrees to site octal output. It has a second entry, OCTO, which sets the initial parameters for OCT with respect to the encoder bit size of the site.

- . MV — This is a double precision subroutine which premultiplies a 3 x 1 vector by a 3 x 3 matrix.

- . TIMEE — This subroutine changes integration time to UTC time and to Local time in days, hours, minutes, and decimal minutes.

- VERNAL — This subroutine computes the mean longitude of the Vernal Equinox at midnight, UTC.

- BLOCK DATA — This subroutine supplies the Besselian day numbers, yearly constants, geodetic constants, and initial values for the program.

2.1 MAIN Program

The following is a description of the pertinent processing flow through the MAIN program. Detailed descriptions of counter loops and miscellaneous computations will be addressed in Appendix III.

A. Initial Input

General input to the program is:

- Number of stations, number of stars per station
- Start day, stop day
- Start hour and stop hour for each day
- Prediction increment size
- Station geodetic and astronomic coordinates
- Special station parameters
- Polar motion parameters †
- Mean coordinates of the stars

First, the station coordinates are read and converted to radians in a right-handed system. Then the site-dependent part of the latitude rotation is computed:

QM (2,2) = DSIN (ALAT) = QM (3,3)

QM (2,3) = DCOS (ALAT) = -QM (3,2)

Where ALAT is the astronomic latitude of the site.

† Polar motion corrections are usually omitted in prediction runs.

Then the diurnal aberration correction factor is computed.

$$AEL = DSQRT(1. - E^2 \, DSIN^2(DLAT)), \text{ and}$$

$$DIURN = 0.3198 * (AEL + HT) * DCOS(DLAT)/FACTOR,$$

where

AEL = radius of earth at the site location (measured in units of Earth semimajor axis),

DIURN = diurnal correction constant,

E = eccentricity of Earth,

HT = Height of site above ellipsoid (measured in units of Earth semimajor axis),

DLAT = geodetic latitude of site, and

FACTOR = conversion factor to convert arcseconds to radians.

Finally, in the call to OCTO, the subroutine OCT is initialized by the bit size of the site encoder.

If polar motion corrections are used, they are read in at this time. The input is:

DUTDOT = First derivative of POL(1,),

POL(1,) = Polar motion time correction (one value for each day of run),

POL(2,) = X-value of polar motion (one value for each day of run), and

POL(3,) = Y-value of polar motion (one value for each day of run).

The initial run parameters (start day, stop day, start hour, stop hour), are converted to UTC time here. If any of the following conditions hold, the run is terminated with an appropriate printout:

a. If start day is a zero,

b. If stop day is later than January 2 of the next year, and

c. If July 1 falls <u>between</u> start day and stop day.

The reason for tests 1 and 2 is that the Besselian day constants only cover one year. The reason behind test 3 is that the Besselian day constants are referenced to the beginning of the nearest Besselian year (about January 1). Hence, they are discontinuous on or about July 1.

B. Daily Corrections and Star Coordinates

At this point the star coordinates are read in. They are:

- mean Right Ascension, epoch at nearest beginning of a Besselian year,
- mean declination, same epoch as Right Ascension,
- absolute parallax,
- proper motion in declination, and
- proper motion in Right Ascension.

The proper motion parameters may be set to zero with little loss of accuracy. The daily parameters are computed now. If the polar motion corrections are used, their effect is also computed here:

```
DAZ = (POL(2, )*DSIN(DLONG) + POL(3, )*DCOS(DLONG))/DCOS(DLAT)

TIM = POL(1, )
```

where

DAZ = azimuth correction,

POL(1,), POL(2,), POL(3,) and DLAT have been previously defined,

DLONG = geodetic longitude of the site, and

TIM = time correction from UTC time to UT1 time.

Provision was made at this point to read in a daily atmosphere for use in sophisticated refraction subroutines.

The next corrections occur in a call to the subroutine CNSTNT. The outputs of this subroutine are the Earth's obliquity (dihedral angle between the plane of the equator and the plane of the Earth's orbit around the sun), and the annual precession in declination - both referenced to the beginning of nearest Besselian year - and the equation of equinoxes and its first derivative, the secular change in the rotation rate of the Earth, and the Besselian day numbers A, B, C, and D along with the Independent day number f, together with their first and second differences, all referenced to 0.0 hour ephemeris time (ET) of the present day. The equations for these parameters will be discussed in the section on the subroutine CNSTNT.

Initially, the Right Ascension and declination are corrected for proper motion and transformed to the X, Y, Z direction cosines (called XV(1), XV(2), and XV(3), respectively). This is a right-handed coordinate system with the X-Y plane coincident with the mean equator and the positive X-axis directed along the mean equinox of the nearest beginning of the Besselian year. The next correction, a second order correction, is computed every ten days, starting with the first day of the run. These corrections are:

DECO = DEC + UD * TTAU

RASO = RAS + UR * TTAU

and

D2DEC = XJP * DTAN (DECO)

D2RAS = XJJ * DTAN (DECO) * DTAN (DECO)

where DEC is the mean declination, UD is the proper motion in declination, RAS is the mean Right Ascension, UR is the proper motion in Right Ascension, TTAU is the fraction of the tropical year which has elapsed from the beginning of the nearest Besselian year, D2DEC is the second order correction in declination, D2RAS is the second order correction in Right Ascension, and XJP and XJJ are constants obtained from subroutine CNSTJ, a second entry into CNSTNT. The formulas for XJP and XJJ will be displayed in the section for the subroutine CNSTNT.

The final parameters determined at this time are the East longitude of the Vernal Equinox at 0.0 hour UT1 and the Earth spin rate with respect to the moving Vernal Equinox. The rate is:

WED = (WE - DW) (1. + DUTDOT), where

WED is the current spin rate,

WE is the spin rate at epoch 1900.0,

DW is the secular decrease in the spin rate as computed in CNSTNT, and

DUTDOT has been previously defined.

C. Diurnal Parallax Corrections

For those stars with a parallax greater than 0.01 in absolute value, the following corrections to the X, Y, Z coordinates are performed.

X1 = -AA(3) * PAR/(CABER * DCOS(E)) - X axis

X2 = -AA(4) * PAR * DCOS(E)/CABER - Y axis

X3 = X2 * DTAN(E) - Z axis

where

PAR is the parallax for this star,

E is the Earth's instantaneous true obliquity (as of 0.0 hour, ET),

CABER = 20.496 arcseconds, and

AA(3) and AA(4) are the Besselian constants C and D, respectively, updated to 0.0 hour, ET, of the present day.

D. Mean-to-Apparent Transformation

The Besselian constants are updated by use of first and second differences via the following formulae.

XL1 = (LT + EPH)/8.64D4

IF(XL1.GT.0.5) HL1 = 1. - XL1

```
C(I) = AA(I) + DA(I) * XL1 + D2A(I) * XL1 * XL1/2.  I = 1,5
IF (XL1.GT.0.5),
C(I) = AAP(I) - DAP(I) * HLI + D2AP(I) * HL1 * HL1/2.  I = 1,5
```

where

LT = time passed since 0.0 hour, UTC, in seconds,

EPH = ET - UTC in seconds,

C(I) = updated Besselian constants,

AA(I), Besselian constants at 0.0 hour, ET,

AAP(I) = same constants, next day,

DA(I) and D2A(I) = first and second differences for this day, and

DAP(I) and D2AP(I) = first and second differences for the next day.

Note that the Besselian constants are interpolated by first and second differences for $\pm$ 1/2 day about 0.0 hour ET.

The next effect is that of annual aberration:

```
XV1(1) = XV(1) - C(4)
XV1(2) = XV(2) + C(3)
XV1(3) = XV(3) + C(3) * DTAN(E)
```

where C(3) and C(4) are the updated Besselian constants C and D, respectively. The other parameters have been previously identified. The precession-nutation effects are accounted for by:

$$\begin{bmatrix} XV2(1) \\ XV2(2) \\ XV2(3) \end{bmatrix} = \begin{bmatrix} 1. & -C(5) & [-C(1) - C(2) * C(5)] \\ C(5) & 1. & [C(2) - C(1) * C(5)] \\ C(1) & -C(2) & 1. \end{bmatrix} \begin{bmatrix} XV1(1) \\ XV1(2) \\ XV1(3) \end{bmatrix}$$

The next effect is the addition of the parallax effect previously calculated:

XV1(1) = XV2(1) + X1

XV1(2) = XV2(2) + X2

XV1(3) = XV2(3) + X3

The coordinates are transformed to the Right Ascension - declination system and corrected for second-order effects previously calculated.

Call ASDCRK (XV1, RAS1, DEC1,1)

RAS1 = RAS1 + D2RAS

DEC1 = DEC1 + D2DEC

The pair RAS1, DEC1 now represent the apparent position of the star.

E. Apparent-to-Observed Transformations

First, the apparent Vernal Equinox and hour angle are computed.

XL11 = LT/8.64D4

EOFE2 = EOFE + XL11 * DEFE

ELNG = ELNGO + EOFE2

TIM1 = TIM

TIM1 = TIM1 + DTDOT * 1D-3 * XL11 if TIM ≠ 0.

DUT = WED * (LT + TIM1)

ANGL = ELNG - DUT

HA = 2. * PI - (RAS1 + ANGL)

where

LT is the time from 0.0 hour, UTC,

EOFE is the equation of equinoxes (difference between apparent and mean equinox) in radians,

DEFE is the change between EOFE now and EOFE tomorrow,

TIM is the difference between UT1 and UTC at 0.0 hour, UTC,

DTDOT is the rate of change of UT1 - UTC in milliseconds,

TIM1 is the updated TIM,

EOFE2 is the updated equation of equinoxes,

ELNG is the updated apparent longitude of the Vernal Equinox without Earth spin,

DUT is the change in the longitude of the Vernal Equinox caused by Earth spin since 0.0 hour, UT1,

ANGL is the updated apparent longitude of the Vernal Equinox, referenced to updated UT1 (if TIM $\neq$ 0.), and

HA is the hour angle of the star.

The diurnal aberration effect can now be removed:

RAS1 = RAS1 + DIURN * DCOS(HA)/DCOS(DEC1)

DEC1 = DEC1 + DIURN * DSIN(HA) * DSIN(DEC1)

where all symbols have already been identified

RAS1 and DEC1 are then transformed to X, Y, Z coordinates.

The instantaneous Right Ascension of the site is then: RA = ALONG - ANGL, where ALONG is the astronomic longitude, thus correcting for deflection of vertical. The star position is then rotated to the site position in longitude and latitude by:

$$\begin{bmatrix} XV(1) \\ XV(2) \\ XV(3) \end{bmatrix} = \begin{bmatrix} 1. & 0. & 0. \\ 0. & DSIN(ALAT) & DCOS(ALAT) \\ 0. & -DCOS(DLAT) & DSIN(ALAT) \end{bmatrix} \begin{bmatrix} -DSIN(RA) & DCOS(RA) & 0. \\ -DCOS(RA) & -DSIN(RA) & 0. \\ 0. & & 1. \end{bmatrix} \begin{bmatrix} XV1(1) \\ XV1(2) \\ XV1(3) \end{bmatrix}$$

Above, ALAT is the site astronomic latitude. Since geocentric aberration can be ignored for stars, there is no transformation along the Earth's radius to the site. Hence:

```
DEL = DSQRT(XV(1)*XV(1) + XV(2)*XV(2))
EL  = DATAN(XV(3)/DEL)†
```

One then checks to see that the EL is larger than the minimum EL for this site. Otherwise, the process restarts at LT + INC seconds later.

The next calculation is AZ:

```
AZ = DATAN2(XV1(1), XV1(2)) * RAD + DAZ
```

Above, AZ is azimuth, EL is elevation, and DAZ is the polar motion correction to azimuth.

Now, if desired, the elevation can be corrected for refraction. The simple correction now used is listed below. This model is surprisingly accurate above 12° elevation, but loses accuracy at low elevation angles. However, a more sophisticated refraction subroutine(s) may easily be added at this point.

```
EL = EL + 3.36D-4 * RAD/DTAN(EL/RAD).
```

† = The computationally faster "EL = DARSIN(XV(3))" was found to be inadequate due to the inaccuracy of the DARSIN subroutine when compared to the DATAN subroutine. See Section 5 for a discussion of this difference.

Plunge and mil outputs are computed via the following equations.

```
AZMIL = AZ * DMIL
ELMIL = EL * DMIL
ELPLG = (180. - EL) * DMIL
AZPLG = DMOD ((AZ + 180.), 360.) * DMIL
```

The octal equivalents of AZ, EL, AZPLG, and ELPLG are computed via a call to OCT.

At this time the printout times are updated via:

```
XMG = XMG + DINC/60DO
XML = XML + DINC/60DO
IF((XMG.GE.60.).OR.(XML.GE.60.).OR.(NQ.EQ.1)) CALL TIMEE
```

where

XMG = number of minutes after the UTC hour,

XML = number of minutes after the local time zone hour,

DINC = integration step size, and

TIMEE is the subroutine which converts the variable LT to days, hours, minutes and decimal minutes in GMT and local time for printout purposes. TIMEE will be discussed in Section 3.

This completes the description of the MAIN program except for various print flags.

3.0 SUBROUTINE PROGRAM DESCRIPTION AND MATHEMATICS

3.1 Subroutine MV

This is a double precision subroutine which premultiplies a 3 x 1 vector by a 3 x 3 matrix. The call is MV(M,V,O) where M is a 3 x 3 matrix with real-valued, double precision entries, V and O are 3 x 1 vectors with real-valued, double-precision entries. M and V are the input, while O is the output.

3.2 Subroutine TIMEE

This is a double precision subroutine which converts elapsed time (seconds from 0.0 hour, UTC) to days, hour, minutes, and decimal minutes in both UTC and local time. This subroutine is called at output time only as all computations are referenced to UTC. The equations are:

LD = UTC day

GH = UTC hour = LT/3600., where LT is the time since 0.0 hour, UTC, in seconds

GH is checked to see that it is not less than 0. nor greater than or equal to 24. If so, LD is increased or decreased by 1 and GH is decreased or increased by 24.

IHG = integral part of GH

MG = (GH - IHG) * 60. = minutes and decimal minutes, UTC

Now local hour is computed. Initially, IDL = LD, where IDL is the local day. Previously, the difference between "local" time and UTC time for each site was computed. For the K^{th} site (the site for which look angles are being computed),

ILOC = integral part of (DLONG(K)-7.5)/15

where DLONG(K) is the West longitude of the site.
ILOC effectively divides the Earth into 24 time zones with meridian of Greenwich centered in zone 0.

Then,

$$\text{IF ZON} \neq 0.,\ \text{ILOC} = \text{ILOC} - 24$$

This equation accounts for the International Date Line and all zones earlier than UTC (i.e., Greenwich).

Finally,

$$\text{TLOC(K)} = \text{ILOC} + \text{TLC}$$

where

TLC = "Legal" local time - "True" local time

See Figure 1 for the "legal" local time zones on the Earth, excluding "daylight savings." Note that the TLC (as read in) can accomodate any time zone as it can be positive or negative and has at least three decimal digits. (See operating instructions in Appendix I.)

Now, the local hour (XLH) is obtained by:

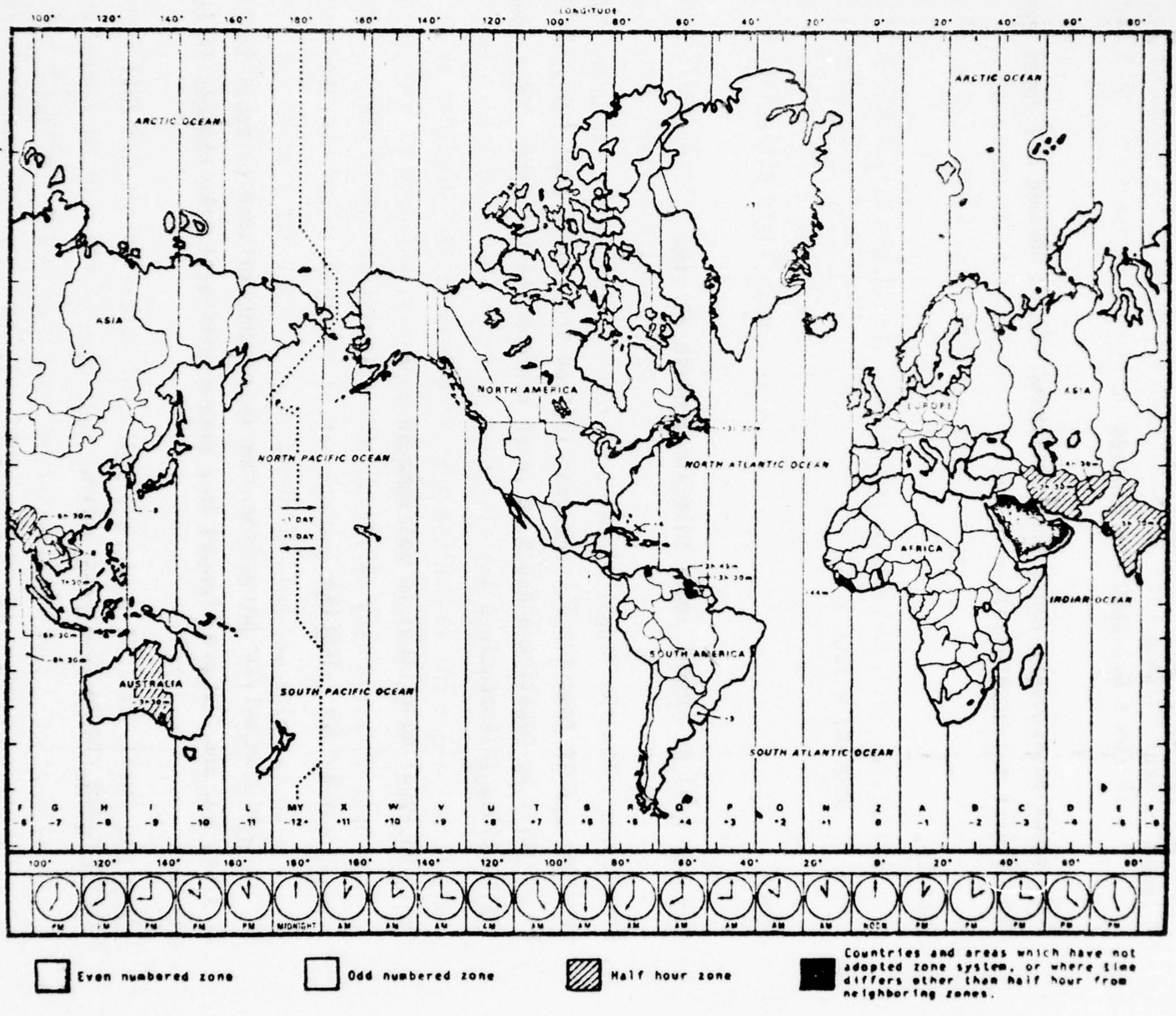
$$\text{XLH} = \text{GH} - \text{TLOC(K)}$$

XLH is then checked for being less than 0. or greater than or equal to 24. If true, IDL and XLH are changed in a manner similar to the changes in LD and GH.

Following the checks discussed above,

Figure 1

STANDARD TIME ZONES OF THE WORLD



IHL = integral part of XLH

ML = (XLH - IHL) * 60

= Minute and decimal minutes of local time (ML is a double precision, real-valued variable.)

The return statement is then reached. The call statement is:

TIMEE(LD,LS,K,AMG,AML),

where the input is:

LD = day of look angle epoch,

LS = elapsed time in seconds from 0.0 hour of LD, and

K = site index (K^{th} site).

The output is:

AMG = minute and decimal minutes of UTC time, and

AML = minutes and decimal minutes of local time.

Also, the labeled common TIME carries IDG, IDL, IHG, IHL, MG, ML, and TLOC(K) from main to the subroutine and back.

3.3 Subroutine Vernal

This subroutine calculates the East longitude of the mean Vernal Equinox at 0.0 hour, UT1, on the given day. This subroutine is taken from Satellite Illumination Prediction Using NORAD 2-Card Element Sets, PA100-74-22. The equations are:

LPYR = integral part of (IYR - 1973)/4, where IYR is the current year

D = 365. * (IYR - 1973.) + LPYR + DAY, where DAY is the current day number

$$D1 = D + 8400.$$

$$ELNG = 260.48673053D0 - 9.85647350 * 10^{-1} * D - 2.9015 * 10^{-13} * D1 * D1$$

ELNG is then converted to an angle less than 360° and greater than or equal to 0°. Then ELNG is converted to radian measure and returned.

ELNG is the East longitude of the Vernal Equinox (sometimes called Point of Aries. In any case, it is the direction of the positive inertial X-axis.) at 0.0 hour, UT1, whose day number is equal to DAY.

The call is VERNAL(IYR,DAY,ELNG). The inputs are DAY and IYR, while the output is ELNG.

3.4 Subroutine Block Data

This subroutine initializes the named commons of TIME, CONST, YR, and INPUT. It is a double precision subroutine.

The common INPUT consists of the variable AI, which has dimension 369 x 5. It contains the Besselian day numbers A, B, C, D and the Independent Day number f as contained in The American Ephemeris and Nautical Almanac for this year. These constants are referenced to the equinox of the beginning of the year until July 1, when the reference becomes the equinox at the beginning of next year. (This "year" is actually the "Besselian year", rather than the "calendar year.") So, in addition to day numbers for every day of the year, there are day numbers for January 0, December 32, and two sets for July 1. Except during leap year, the 369th entries are zero for all five variables. All of these numbers are supplied by data statements in this subroutine.

The common TIME has the variables IDG, IDL, IHG, IHL, XMG, XML, TIM, and TLOC. TLOC has dimension 10. TIM is described in the Section on Main, while the others are described in the secion on the subroutine TIMEE. Block data is used to initialize IDG, IDL, IHG, IHL, XMG, and XML to 0.

The common YR has the data for the current year. The variables are XMJYR, IYR, NYR, and July. XMJYR is the modified Julian date of January 0 of the current

year (December 31 of last year). IYR is the year number of the current year, and NYR is the number of days in this year. JULY is the day number (day) of July 1 for the current year. All of these variables are initialized via data statements in BLOCK DATA.

The last common, CONST, contains most of the Earth model (WGS-72), time, and numerical constants used in this program. They are:

- PI - the constant π (=3.141592653598732D0),
- RAD - the number of degrees in a radian (=57.295779513082D0),
- FILM - the number of meters in an International foot (=3.048D-1),
- EC2 - the eccentricity squared of the Earth through the poles (=6.694317778D-3),
- ES - the mean current eccentricity of the Earth's orbit (=1.6726D-2),
- FLAT - the flattening constant of the Earth (=298.26D0),
- AE - the average radius of the Earth in meters (=6378135D0),
- WE - the average spin velocity of the Earth with respect to the (moving) mean Vernal Equinox in radians/second (=7.29211585468D-5) for epoch 1900.0,
- CABER - the aberrational constant of the Sun (=20.496D0), and
- EPH - the current difference between Ephemeris time and UTC (ET - UTC) in seconds. (EPH can be found on page *vii* of the American Ephemeris and Nautical Almanac for the year desired.)

3.5 Subroutine ASDCRK

This is a double precision subroutine which converts Right Ascension and declination to X, Y, Z direction cosines and vice versa. The flag input is N. If N = 1, X-Y-Z cosines are converted to Right Ascension and declination. If N = 0, then Right Ascension and declination are converted to X-Y-Z cosines.

The call is ASDCRK(X,RAS,DEC,N), where X is a vector of dimension 3. If N = 1, X is the input, and RAS and DEC are the outputs (RAS is Right Ascension, and DEC is declination.).

Otherwise, RAS and DEC are the inputs while X is the output. N is always an input.

3.6 Subroutine CNSTNT

This is a double precision subroutine which computes the various constants needed in the program. Those constants are:

E = Earth's mean obliquity (dihedral angle between the equatorial plane and Earth's orbital plane - called the ecliptic plane). This parameter is always referenced to the beginning of the nearest Besselian year via use of the flag LQ.

XMO = annual general precession in declination, referenced to the beginning of the Besselian year (similar to E).

The equations for these parameters are:

```
TAU = IYR - 1900.
IF(LQ.EQ.0) GOTO 3
DB = 15019.81352 + (365.24219879 - 8.56 * 1D-9 * TAU) * TAU - XMJYR
DL  = 365.2422 - 1.48D-3 * TAU/8.64D-4
TTO = (XMJYR - 15019.499995)/36525D0
DAY = XMJYR - 15019.81352 + DB
IF(J.GE.JULY) DAY = DL + DAY
DAY1 = DAY + 3.1352D-1 + 5.D-6
TT = DAY/36525.
TET = DAY1/36525.
E  = (23.4522944 - 1.30125D-2 * TET - 1.639D-6 * TET * TET
     + 5.028D-7 * (TET **3))/RAD
```

```
      XMO = (20.04688D0 - 8.50 D-3 * TT)/FACTOR
      ETN = -DTAN(E)
      LQ  = 0
    3 CONTINUE
```

The flag LQ is reset to LQ = 1 for each run. Above,

DB = days and decimal days from 0.0 hour, ET, January 0, of the current year to the beginning of the current Besselian year,

DL = length of current Besselian year,

XMJYR = Modified Julian date of January 0 for the current year,

TTO = fraction of a computational tropical century which has elapsed since the beginning of the Ephermeris Julian year of 1900,

TT = fraction of a computational tropical century which has elapsed since the beginning of the Ephemeris Besselian year of 1900,

TET = TO,

UTC, E, XMO, and LQ were previously defined.

Now the Besselian constants for the current day (day number J) are taken from Block Data and transferred to the variables AA, DA, and D2A - all of dimension 5. DA and D2A contain the first and second differences, respectively, of the Besselian constants. All of these variables are in radians, radians/day, or radians/day^2, as required. The equations are:

```
      J1 = J + 1
      IF (J.GE. July) J1 = J1 + 1
      AA(I) = AI(J1,I)/Factor; I = 1,5
```

Now let "0" represent the current day, "-1" yesterday, and "1" tomorrow.

Then:

```
      DA(0)  = AA(1) - AA(0)
      D2A(0) = AA(1) + AA(-1) - 2. * AA(0) for the five AA( )'s
```

Also, AA(I), and D2A(I) are calculated for day J + 1 and named AAP, and D2AP, respectively, but DAP = DA.

These constants are used in the following manner. Let XL = fraction of day J for which look angles are calculated. Then:

```
C(I) = AA(I) + DA(I) * XL + D2A(I) * XL * XL/2., I = 1,5
```

if XL is less than or equal to 1/2. Otherwise,

```
C(I) = AAP(I) - DAP(I) * (1 - XL) +
        D2AP(I) * (1 - XL) * (1 - XL)/2., I = 1,5
```

This computation is accomplished in the MAIN program.

Due to the discontinuity at July 1 and the end of the year, the AAP, DAP, and D2AP vectors are reset for those two days.

```
    IF ((J.NE.(July - 1).AND.(J.NE.NYR)) GOTO 2
    AAP(I)  = AA(I)
    DAP(I)  = -DA(I)
    D2AP(I) = D2A(I)
  2 CONTINUE.
```

Also, in MAIN, XL is used past XL = 1/2 instead of using 1 - XL.

Finally, the parameters EOFE, DEFE, and DW are calculated. These are:

EOFE = "Equation of Equinoxes" (difference between mean and apparent Vernal Equinox at 0.0 hour, UTC),

DEFE = difference in EOFE from day J to day J+1, and

DW = secular change in Earth spin rate.

The equations are:

```
TT1    = TTO + J/36525DO
TTAU   = (J - DB)/365.2422DO
TTAUI  = TTAU + 1./365.2422DO
IF (J.GE.JULY) TTAU = TTAU - DL/365.2422DO
IF (J.GE.JULY) TTAU1 = TTAU1 - DL/365.2422DO
XM     = XMO * TTAU
XM1    = XMO * TTAUI
EOFE   = (AA(1) - XM)/ETN
EOFE1  = (AAP(1) - XM1)/ETN
DEFE   = EOFE1 - EOFE
DW     = 4.28 * 1D-15 * TT1
```

Note that AA(1) is the Besselian Day Number A.

The call to CNSTNT is CNSTNT(J,E,DW,EOFE,DEFE).
The only input here is J, the day number. Other inputs obtained from common are:

Common CONST - the constant RAD,

Common YR - the yearly constants,

Common INPUT - the Besselian Day Numbers, and

Common (unnamed) - the variables FACTOR, DSN1, and LQ.

FACTOR is the number of arcseconds in a radian, while DSN1 is the double precision sine of an arcsecond.

Outputs are E, DW, EOFE, and DEFE. Also, the <u>current</u> Besselian Day Numbers and those for the succeeding day, together with their first and second differences, are passed through the named common NOW. The fraction of a tropical year which has elapsed (will elapse) since the beginning of the nearest Besselian year, TTAU, is passed through the unnamed common.

There is a second entry to CNSTNT, called CNSTJ. The call is CNSTJ(J,IJ,RAS,DEC,XJJ,XJP). The inputs are:

J - the current day number;

IJ - set in MAIN, IJ = -1 if the star declination is negative, IJ = +1 otherwise;

RAS - updated mean Right Ascension of star; and

DEC - updated mean declination of star.

The outputs (in units of radians) are:

XJJ - the 2^{nd} order correction to Right Ascension, and

XJP - the 2^{nd} order correction to declination.

The equations are:

```
P1  = (A(J)+IJ*D(J))*DSIN(RAS)+(B(J)+IJ*C(J))*DCOS(RAS)
P2  = (A(J)+IJ*D(J))*DCOS(RAS)-(B(J)+IJ*C(J))*DSIN(RAS)
XJJ = P1*P2*DSN1/FACTOR
XJP = -P1*P1*DSN1/(2.*FACTOR)
```

Above, A(J), B(J), C(J), and D(J) are the Besselian day numbers A, B, C, and D for day J.

3.7 Subroutine OCT

This subroutine changes degrees into octal encoder readout. Some explanation of encoder readout is required. The encoder has a set number of "bits" - say, 17. Each bit is binary (either on or off). The first bit (reading from left to right) is "1" if the angle is above 180°. Otherwise, it is "0". The second bit is "1" if the angle (or the remainder of angle -180° if bit 1 = 1) is above 90° (90 = 180/2), otherwise it is "0". The third bit is "1" if the angle (for the appropriate remainder) is above 45° (45 = 90/2) and is "0" otherwise. This process continues down to the least significant bit (LSB). In this case, the 17th bit = 9.89 arcseconds. See Table 1A for a short summary of LSB values for popular bit configurations.

TABLE 1A

Short Table of LSB's for Common Angle Encoder Bit Configurations

BIT #	LSB ARCSECONDS	MILS
13	158.202	0.7825
15	39.548	0.19531
17	9.889	0.04883
19	2.473	0.01221
21	0.619	0.00305
23	0.155	0.00076

After determining the binary bit number, every set of three starting with the LSB is changed to octal. As an example, let us change the following angle to 17 bit octal representation.

TABLE 1B

Computation of Binary 17-Bit Representation

Angle = 123.426°

BIT #	VALUE	REMAINDER
1	0	123.426
2	1	43.426
3	0	43.426
4	1	20.926
5	1	9.676
6	1	4.051
7	1	1.2385
8	0	1.2385
9	1	0.535375
10	1	0.183812
11	1	0.008031
12	0	0.008031
13	0	0.008031
14	0	0.008031
15	0	0.008031
16	1	0.002538
17	0	0.002538

The binary value, in groups of 3, is: 01 011 110 111 000 010. The octal representation of that number is 136702.

The process as shown above is similar to the process used in OCT. First, there is a second entry called OCTO. The call is OCTO(IBITE). The input is IBITE, the number of encoder bits, read from the station card. If IBITE = 0, this call is skipped. In OCTO the number of groups of three are determined, as are the number of bits in the first set (3, 2, or 1). The division for the first group of three is thus determined (180° if the number in the first set is 3, 90° if it is 1, and 45° if it is 2). Several other flags are also set to be used when OCT is called.

The call for OCT is OCT(X,IOUT), where X is the input - a double precision number in decimal degrees - and IOUT is the octal (integer) representation. The heart of OCT is the double DO loop shown below.

```
     DO 6 K = LO, L
     DO 5 J = 1,3
     R = 2.**J
     IF(XF.GE.(DIV/R))  IBIT(K) = IBIT(K) + (8./R)
   5 IF(XF.GE.(DIV/R)) XF = XF - (DIV/R)
   6 DIV = DIV/8D0
```

Above, XF is the remainder passed into this loop after processing the odd number of front-end bits. DIV, LO, and L are set in OCTO. The vector IBIT stores the result. IBIT is zeroed on entry into OCT.

After this processing is complete, the remainder is checked to see if it is greater or equal to three-fourths the LSB. If so, the last octal number is increased by 1. (The number "3/4" is an "engineering" choice based on the inherent noise in the LSB of most encoders. A mathematical choice would be "1/2".)

If the smallest octal integer is increased, it is checked to see if it is 8. If so, it is zeroed and the next smallest integer is increased by one. This continues until an integer is reached which is not 8 after increase or until the first integer is increased. It is then checked to see if the number is equivalent to zero in octal representation. If so, the entire number is zeroed. Figure 2 shows an example of many types of octal output. Figure 3 shows four examples in the Azimuth Octal column of the roundup portion of the subroutine.

Finally, the single-digit integers in the vector IBIT are converted into a multi-digit integer. This number is IOUT. IF X, the input, is negative, then IOUT is multiplied by -1.

Figure 2

Different Samples of Octal Printout .

13 Bit Encoder

ZDAY	LDAY	UTC HR MIN	LOCAL HR MIN	AZIMUTH DEG	ELEVATION DEG	AZIMUTH MILS	ELEVATION MILS	PLUNGAZ MILS	PLUNGEL MILS	AZIMUTH OCTAL	ELEVATION OCTAL	PLUNGAZ OCTAL	PLUNGEL OCTAL
182	182	8:30.00	0:30.0	67.53608	1.39258	1200.641	24.757	4400.64	3175.24	3001	37	[illegible]	7740
182	182	9: 0.0	1: 0.0	71.64077	7.19854	1273.614	127.974	4473.61	3072.03	3136	244	[illegible]	7534
182	182	9:30.00	1:30.0	75.59417	13.14172	1343.896	233.631	4543.90	2966.37	3270	453	[illegible]	7325
182	182	10: 0.0	2: 0.0	79.47330	19.19018	1412.859	341.159	4612.86	2858.84	3420	664	13420	7113
182	182	10:30.00	2:30.0	83.36182	25.31518	1481.988	450.048	4681.99	2749.95	3551	1100	13551	6700
182	182	11: 0.0	3: 0.0	87.35770	31.48912	1553.026	559.807	4753.03	2640.19	3704	1314	13704	6463
182	182	11:30.00	3:30.0	91.58539	37.68305	1628.185	669.921	4828.18	2530.08	4044	1531	14044	6246

17 Bit Encoder

182	182	8:30.00	0:30.0	67.53608	1.39258	1200.641	24.757	4400.64	3175.24	60015	773	260015	177005
182	182	9: 0.0	1: 0.0	71.64077	7.19854	1273.614	127.974	4473.61	3072.03	62743	5075	262743	172703
182	182	9:30.00	1:30.0	75.59417	13.14172	1343.896	233.631	4543.90	2966.37	65603	11261	265603	166517
182	182	10: 0.0	2: 0.0	79.47330	19.19018	1412.859	341.159	4612.86	2858.84	70407	15513	270407	162265
182	182	10:30.00	2:30.0	83.36182	25.31518	1481.988	450.048	4681.99	2749.95	73217	22001	273217	155777
182	182	11: 0.0	3: 0.0	87.35770	31.48912	1553.026	559.807	4753.03	2640.19	76076	26311	276076	151467
182	182	11:30.00	3:30.0	91.58539	37.68305	1628.185	669.921	4828.18	2530.08	101101	32630	301101	145150

19 Bit Encoder

182	182	8:30.00	0:30.0	67.52459	1.38491	1200.437	24.621	4400.44	3175.38	300044	3741	1300044	774037
182	182	9: 0.0	1: 0.0	71.62949	7.19044	1273.413	127.830	4473.41	3072.17	313576	24350	1313576	753430
182	182	9:30.00	1:30.0	75.58294	13.13328	1343.697	233.481	4543.70	2966.52	326773	45266	1326773	732511
182	182	10: 0.0	2: 0.0	79.46194	19.18148	1412.657	341.004	4612.66	2859.00	342015	66437	1342015	711341
182	182	10:30.00	2:30.0	83.35012	25.30629	1481.780	449.890	4681.78	2750.11	355053	107767	1355053	670011
182	182	11: 0.0	3: 0.0	87.34540	31.48013	1552.807	559.647	4752.81	2640.35	370346	131426	1370346	646351
182	182	11:30.00	3:30.0	91.57215	37.67404	1627.949	669.761	4827.95	2530.24	404361	153123	1404361	624655

23 Bit Encoder

182	182	8:30.00	0:30.0	67.53608	1.39258	1200.641	24.757	4400.64	3175.24	6001510	77301	26001510	17700476
182	182	9: 0.0	1: 0.0	71.64077	7.19854	1273.614	127.974	4473.61	3072.03	6274347	507472	26274347	17270306
182	182	9:30.00	1:30.0	75.59417	13.14172	1343.896	233.631	4543.90	2966.37	6560300	1126060	26560300	16651717
182	182	10: 0.0	2: 0.0	79.47330	19.19018	1412.859	341.159	4612.86	2858.84	7040726	1551273	27040726	16226504
182	182	10:30.00	2:30.0	83.36182	25.31518	1481.988	450.048	4681.99	2749.95	7321707	2200076	27321707	15577701
182	182	11: 0.0	3: 0.0	87.35770	31.48912	1553.026	559.807	4753.03	2640.19	7607576	2631065	27607576	15146712
182	182	11:30.00	3:30.0	91.58539	37.68305	1628.185	669.921	4828.18	2530.08	10110116	3262776	30110116	14515001

IS STATION SITE 1 LAT = 37.5 DEGS LOCAL TIME DIFF FROM GMT =+ 8 0 HRS.

FIGURE 3
Examples of Round-up

ALFA BETA

No OCTAL Round-up

DAY	UTC HR MIN	LOCAL HR MIN	AZIMUTH DEG	ELEVATION DEG	AZIMUTH MILS	ELEVATION MILS	PLUNGAZ MILS	PLUNGEL MILS	AZIMUTH OCTAL	ELEVATION OCTAL	PLUNGAZ OCTAL	PLUNGEL OCTAL
179	8:26.38	0:26.4	359.93427	3.89976	6398.832	69.329	3198.83	3130.67	17776	130	7776	7647
179	8:26.42	0:26.4	359.93891	3.89975	6398.914	69.329	3198.91	3130.67	17776	130	7776	7647
179	8:26.45	0:26.4	359.94354	3.89974	6398.996	69.329	3199.00	3130.67	17776	130	7776	7647
179	8:26.48	0:26.5	359.94818	3.89974	6399.079	69.329	3199.08	3130.67	17776	130	7776	7647
179	8:26.52	0:26.5	359.95281	3.89973	6399.161	69.329	3199.16	3130.67	17776	130	7776	7647
179	8:26.55	0:26.5	359.95744	3.89973	6399.243	69.328	3199.24	3130.67	17777	130	7777	7647
179	8:26.58	0:26.6	359.96208	3.89972	6399.326	69.328	3199.33	3130.67	17777	130	7777	7647
179	8:26.62	0:26.6	359.96671	3.89972	6399.408	69.328	3199.41	3130.67	17777	130	7777	7647
179	8:26.65	0:26.6	359.97135	3.89972	6399.491	69.328	3199.49	3130.67	17777	130	7777	7647
179	8:26.68	0:26.7	359.97598	3.89971	6399.573	69.328	3199.57	3130.67	17777	130	7777	7647
179	8:26.72	0:26.7	359.98062	3.89971	6399.655	69.328	3199.66	3130.67	17777	130	7777	7647
179	8:26.75	0:26.7	359.98525	3.89971	6399.738	69.328	3199.74	3130.67	17777	130	7777	7647
179	8:26.78	0:26.8	359.98989	3.89971	6399.820	69.328	3199.82	3130.67	17777	130	7777	7647
179	8:26.82	0:26.8	359.99452	3.89971	6399.903	69.328	3199.90	3130.67	17777	130	7777	7647
179	8:26.85	0:26.8	359.99915	3.89971	6399.985	69.328	3199.98	3130.67	17777	130	7777	7647
179	8:26.88	0:26.9	0.00379	3.89971	0.067	69.328	3200.07	3130.67	0	130	10000	7647
179	8:26.92	0:26.9	0.00842	3.89971	0.150	69.328	3200.15	3130.67	0	130	10000	7647
179	8:26.95	0:26.9	0.01306	3.89971	0.232	69.328	3200.23	3130.67	0	130	10000	7647
179	8:26.98	0:27.0	0.01769	3.89971	0.315	69.328	3200.31	3130.67	0	130	10000	7647
179	8:27.02	0:27.0	0.02233	3.89971	0.397	69.328	3200.40	3130.67	0	130	10000	7647

OCTAL Round-up, No 360° Protection

DAY	UTC HR MIN	LOCAL HR MIN	AZIMUTH DEG	ELEVATION DEG	AZIMUTH MILS	ELEVATION MILS	PLUNGAZ MILS	PLUNGEL MILS	AZIMUTH OCTAL	ELEVATION OCTAL	PLUNGAZ OCTAL	PLUNGEL OCTAL
179	8:26.38	0:26.4	359.93427	3.89976	6398.832	69.329	3198.83	3130.67	17776	130	7776	7647
179	8:26.42	0:26.4	359.93891	3.89975	6398.914	69.329	3198.91	3130.67	17776	130	7776	7647
179	8:26.45	0:26.4	359.94354	3.89974	6398.996	69.329	3199.00	3130.67	17776	130	7776	7647
179	8:26.48	0:26.5	359.94818	3.89974	6399.079	69.329	3199.08	3130.67	17777	130	7777	7647
179	8:26.52	0:26.5	359.95281	3.89973	6399.161	69.329	3199.16	3130.67	17777	130	7777	7647
179	8:26.55	0:26.5	359.95744	3.89973	6399.243	69.328	3199.24	3130.67	17777	130	7777	7647
179	8:26.58	0:26.6	359.96208	3.89972	6399.326	69.328	3199.33	3130.67	17777	130	7777	7647
179	8:26.62	0:26.6	359.96671	3.89972	6399.408	69.328	3199.41	3130.67	17777	130	7777	7647
179	8:26.65	0:26.6	359.97135	3.89972	6399.491	69.328	3199.49	3130.67	17777	130	7777	7647
179	8:26.68	0:26.7	359.97598	3.89971	6399.573	69.328	3199.57	3130.67	17777	130	7777	7647
179	8:26.72	0:26.7	359.98062	3.89971	6399.655	69.328	3199.66	3130.67	17777	130	7777	7647
179	8:26.75	0:26.7	359.98525	3.89971	6399.738	69.328	3199.74	3130.67	17777	130	7777	7647
179	8:26.78	0:26.8	359.98989	3.89971	6399.820	69.328	3199.82	3130.67	20000	130	10000	7647
179	8:26.82	0:26.8	359.99452	3.89971	6399.903	69.328	3199.90	3130.67	20000	130	10000	7647
179	8:26.85	0:26.8	359.99915	3.89971	6399.985	69.328	3199.98	3130.67	20000	130	10000	7647
179	8:26.88	0:26.9	0.00379	3.89971	0.067	69.328	3200.07	3130.67	0	130	10000	7647
179	8:26.92	0:26.9	0.00842	3.89971	0.150	69.328	3200.15	3130.67	0	130	10000	7647
179	8:26.95	0:26.9	0.01306	3.89971	0.232	69.328	3200.23	3130.67	0	130	10000	7647
179	8:26.98	0:27.0	0.01769	3.89971	0.315	69.328	3200.31	3130.67	0	130	10000	7647
179	8:27.02	0:27.0	0.02233	3.89971	0.397	69.328	3200.40	3130.67	0	130	10000	7647

Full OCTAL Round-up, Standard Round-up Parameter (=3/4)

DAY	UTC HR MIN	LOCAL HR MIN	AZIMUTH DEG	ELEVATION DEG	AZIMUTH MILS	ELEVATION MILS	PLUNGAZ MILS	PLUNGEL MILS	AZIMUTH OCTAL	ELEVATION OCTAL	PLUNGAZ OCTAL	PLUNGEL OCTAL
179	8:26.38	0:26.4	359.93427	3.89976	6398.832	69.329	3198.83	3130.67	17776	130	7776	7647
179	8:26.42	0:26.4	359.93891	3.89975	6398.914	69.329	3198.91	3130.67	17776	130	7776	7647
179	8:26.45	0:26.4	359.94354	3.89974	6398.996	69.329	3199.00	3130.67	17776	130	7776	7647
179	8:26.48	0:26.5	359.94818	3.89974	6399.079	69.329	3199.08	3130.67	17777	130	7777	7647
179	8:26.52	0:26.5	359.95281	3.89973	6399.161	69.329	3199.16	3130.67	17777	130	7777	7647
179	8:26.55	0:26.5	359.95744	3.89973	6399.243	69.328	3199.24	3130.67	17777	130	7777	7647
179	8:26.58	0:26.6	359.96208	3.89972	6399.326	69.328	3199.33	3130.67	17777	130	7777	7647
179	8:26.62	0:26.6	359.96671	3.89972	6399.408	69.328	3199.41	3130.67	17777	130	7777	7647
179	8:26.65	0:26.6	359.97135	3.89972	6399.491	69.328	3199.49	3130.67	17777	130	7777	7647
179	8:26.68	0:26.7	359.97598	3.89971	6399.573	69.328	3199.57	3130.67	17777	130	7777	7647
179	8:26.72	0:26.7	359.98062	3.89971	6399.655	69.328	3199.66	3130.67	17777	130	7777	7647
179	8:26.75	0:26.7	359.98525	3.89971	6399.738	69.328	3199.74	3130.67	17777	130	7777	7647
179	8:26.78	0:26.8	359.98989	3.89971	6399.820	69.328	3199.82	3130.67	0	130	10000	7647
179	8:26.82	0:26.8	359.99452	3.89971	6399.903	69.328	3199.90	3130.67	0	130	10000	7647
179	8:26.85	0:26.8	359.99915	3.89971	6399.985	69.328	3199.98	3130.67	0	130	10000	7647
179	8:26.88	0:26.9	0.00379	3.89971	0.067	69.328	3200.07	3130.67	0	130	10000	7647
179	8:26.92	0:26.9	0.00842	3.89971	0.150	69.328	3200.15	3130.67	0	130	10000	7647
179	8:26.95	0:26.9	0.01306	3.89971	0.232	69.328	3200.23	3130.67	0	130	10000	7647
179	8:26.98	0:27.0	0.01769	3.89971	0.315	69.328	3200.31	3130.67	0	130	10000	7647
179	8:27.02	0:27.0	0.02233	3.89971	0.397	69.328	3200.40	3130.67	0	130	10000	7647

Full OCTAL Round-up Parameter = 1/2

DAY	UTC HR MIN	LOCAL HR MIN	AZIMUTH DEG	ELEVATION DEG	AZIMUTH MILS	ELEVATION MILS	PLUNGAZ MILS	PLUNGEL MILS	AZIMUTH OCTAL	ELEVATION OCTAL	PLUNGAZ OCTAL	PLUNGEL OCTAL
179	8:26.38	0:26.4	359.93427	3.89976	6398.832	69.329	3198.83	3130.67	17777	131	7777	7647
179	8:26.42	0:26.4	359.93891	3.89975	6398.914	69.329	3198.91	3130.67	17777	131	7777	7647
179	8:26.45	0:26.4	359.94354	3.89974	6398.996	69.329	3199.00	3130.67	17777	131	7777	7647
179	8:26.48	0:26.5	359.94818	3.89974	6399.079	69.329	3199.08	3130.67	17777	131	7777	7647
179	8:26.52	0:26.5	359.95281	3.89973	6399.161	69.329	3199.16	3130.67	17777	131	7777	7647
179	8:26.55	0:26.5	359.95744	3.89973	6399.243	69.328	3199.24	3130.67	17777	131	7777	7647
179	8:26.58	0:26.6	359.96208	3.89972	6399.326	69.328	3199.33	3130.67	17777	131	7777	7647
179	8:26.62	0:26.6	359.96671	3.89972	6399.408	69.328	3199.41	3130.67	17777	131	7777	7647
179	8:26.65	0:26.6	359.97135	3.89972	6399.491	69.328	3199.49	3130.67	17777	131	7777	7647
179	8:26.68	0:26.7	359.97598	3.89971	6399.573	69.328	3199.57	3130.67	17777	131	7777	7647
179	8:26.72	0:26.7	359.98062	3.89971	6399.655	69.328	3199.66	3130.67	0	131	10000	7647
179	8:26.75	0:26.7	359.98525	3.89971	6399.738	69.328	3199.74	3130.67	0	131	10000	7647
179	8:26.78	0:26.8	359.98989	3.89971	6399.820	69.328	3199.82	3130.67	0	131	10000	7647
179	8:26.82	0:26.8	359.99452	3.89971	6399.903	69.328	3199.90	3130.67	0	131	10000	7647
179	8:26.85	0:26.8	359.99915	3.89971	6399.985	69.328	3199.98	3130.67	0	131	10000	7647
179	8:26.88	0:26.9	0.00379	3.89971	0.067	69.328	3200.07	3130.67	0	131	10000	7647
179	8:26.92	0:26.9	0.00842	3.89971	0.150	69.328	3200.15	3130.67	0	131	10000	7647
179	8:26.95	0:26.9	0.01306	3.89971	0.232	69.328	3200.23	3130.67	0	131	10000	7647
179	8:26.98	0:27.0	0.01769	3.89971	0.315	69.328	3200.31	3130.67	0	131	10000	7647
179	8:27.02	0:27.0	0.02233	3.89971	0.397	69.328	3200.40	3130.67	0	131	10000	7647

4.0 EFFECT OF CORRECTIONS AND APPROXIMATIONS

In this section the effect of the various corrections to the mean star position (transformation from "mean" to "apparent") and to the apparent star position (transformation from "apparent" to "look angles") will be discussed. This section consists of self comparisons of the verified program. Its purpose is to establish a basis for estimating the numerical "size" of various corrections before discussing verification efforts. These comparison were made by use of the "degrees" output, which has a LSB of 0.00001 degrees or 0.036 arcseconds. Hence, the tables in this report only show changes of about 0.04 arcseconds or more. This step size is only in the printout and not in the computer processing.

4.1 Transformation from "MEAN" to "APPARENT"

The transformations here, in order of use, are:

a. Mean motion is corrected for "proper motion" with respect to the mean equator and equinox of the nearest Besselian New Year;

b. The rectangular coordinates are corrected for "annual" aberration;

c. The rectangular coordinates are transformed to the "true" equinox and "true" equator of date (precession-nutation corrections);

d. The annual parallax correction (transformation from heliocen - tric to geodetic coordinates) is introduced; and

e. Second order corrections caused by cross-terms involving aberration corrections for a fixed star system and precession - nutation corrections for a moving star system are introduced.

The assumptions involved are:

a. No star has a declination of exactly $\pm$ 90°.

b. Second order proper motion corrections are ignored. This is tenable because the largest corrections used here cover only 1/2 year, while second order proper motion corrections are on the order of 0.01 arcsecond/tropical century.

c. "Small angle" approximation ($\sin(X) = X$ and $\cos(X) = 1.-X^2/2.$, where X is in radians and all second order terms are omitted in the final computation) is used throughout this program, particularly in the precession-nutation computation. Replacement of these approximations by actual sines and cosines resulted in no difference to the order desired (5×10^{-3} arcseconds).

d. The parallax correction is omitted if $\pi < 0.01$ arcseconds. Also, this correction is assumed constant over a day's period. Again, the error is less than 5×10^{-3} arcseconds.

e. One can also introduce second order errors for the precession-nutation transformation (see reference 1, p. 370). However, there is no evidence that this is required for 0.01 arcsecond accuracy.

f. Corrections for the Earth's elliptic terms are not used. These corrections amount to 0.001 arcsecond/tropical century and are not significant.

g. Additional assumptions are:

1. The motion of the Earth lies in the ecliptic;

2. Second order effects of aberration are negligible; and

3. "Light" time and "secular" aberration are never corrected (for stars).

The following effects are arranged in order of decreasing effect. The process followed is to compare outputs run from a deck with all effects "zeroed out" against a program run with all effects "zeroed out" except the stated effect. The effects are:

Diurnal aberration;

Update of Besselian constants by first and second differences;

Update of Besselian constants about a period of [0,1] instead of a period of [-1/2, 1/2] - the fractions refer to "days";

Update of Besselian constants using first differences only;

Linear update of Equation of Equinoxes;

Second order calculations (J and J' numbers); and

Secular change to Earth spin rate.

4.2 Diurnal Aberration

The order of all the tables will be "corrected - uncorrected." Table 3 below is "Diurn Corrected" - "Std." Table 2 lists the input data, which is the same for the following tables, except for day of run. For Table 3 the "rough" azimuth and elevation for the "std" run are listed for help in interpreting the data. The other tables for this same day would have the same "rough" pointing angles.

TABLE 2

Input Data for Runs Used in Section 4

Latitude (site) = 37.4958° N

Longitude (site) = 237.4961° E

Height above ellipsoid = 0. ft.

Astronomic latitude - Geodetic latitude = 0.

Astronomic longitude - Geodetic longitude = 0.

Minimum printed elevation = 0.°

Star Name = ALFA BETA†

Star Mean declination = 56°24'22"

Star Mean Right Ascension = 99°47'1.5"(6 hours,39 min,8.1 sec)

Star Epoch = 1976.0

Month and Day of Run = 1/1 GMT

Note that the DIURN parameter (in arcseconds) is:

DIURN = 3.198D-1*(AEL+HT)*DCOS(DLAT),

where (AEL + HT) is the distance (in standard Earth radii) from the site to the Earth's center and DLAT is the geodetic latitude. This parameter has a maximum of 0.32 arcseconds for HT = 0 ft. and DLAT = 0°.

† Not listed in most star catalogs.

TABLE 3

Diurnal Aberration Test

Time GMT	Az degrees	Azimuth Diff. arcseconds	El degrees	Elevation Diff. arcseconds
1:00-1/1	35.	-.18	22.	-.18
2:00	39.	-.22	9.	-.14
3:00	43.	-.22	37.	-.14
4:00	44.	-.25	45.	-.14
5:00	43.	-.32	54.	-.07
6:00	38.	-.40	61.	-.11
7:00	26.	-.47	68.	-.11
8:00	4.	-.43	71.	-.18
9:00	340.	-.22	69.	-.22
10:00	325.	-.07	64.	-.25
11:00	318.	0.	56.	-.25
12:00	316.	-.04	48.	-.25
13:00	317.	0.	39.	-.25
14:00	320.	-.04	31.	-.25
15:00	324.	-.04	24.	-.22
16:00	329.	-.04	17.	-.22
17:00	337.	-.07	12.	-.22
18:00	343.	-.07	8.	-.18
19:00	351.	-.14	5.	-.18
20:00	359.	-.11	4.	-.18
21:00	7.	-.14	5.	-.18
22:00	15.	-.18	7.	-.18
23:00	23.	-.18	11.	-.18
0:00-1/2	29.	-.18	16.	-.18
1:00	35.	-.18	22.	-.14

4.3 Update of Besselian Constants

The second effect studied is the use of first and second differences to update the Besselian constants. In Table 4 the differences between "usual" - "no update" are recorded. Note that these differences are large enough to <u>require</u> interpolation for high accuracy work.

TABLE 4

Test for Full Updating of Besselian Constants

Time GMT	Azimuth Diff. arcseconds	Elevation Diff. arcseconds
1:00-1/1	0.	0.
2:00	-.04	0.
3:00	-.04	-.04
4:00	-.04	-.04
5:00	-.07	-.04
6:00	-.04	-.07
7:00	+.04	-.07
8:00	+.18	-.11
9:00	+.29	-.04
10:00	+.29	+.04
11:00	+.22	+.04
12:00	+.14	+.11
13:00	+.11	+.14
14:00	+.04	+.18
15:00	+.04	+.18
16:00	0.	+.18
17:00	-.04	+.18
18:00	-.07	+.22
19:00	-.14	+.18
20:00	-.18	+.18
21:00	-.22	+.14
22:00	-.25	+.11
23:00	-.29	+.04
0:00-1/2	0.	0.
1:00	0.	0.

Below, Table 5 records the differences between the look-angles produced by the usual update method (update interval being [-1/2, + 1/2) in terms of days about the midnight of the interpolated day) to those produced by use of an interpolation period of [0,1). The differences are "[0,1) period" - "usual". Table 6 records the differences between the usual method (first and second differences) and the values produced by use of first differences only. Again, the differences are recorded in the sense "first only" - "usual."

TABLE 5

Test for Updating of Besselian Constants,

Change of Usual Interpolation Interval

Time GMT	Azimuth Diff. arcseconds	Elevation Diff. arcseconds
1:00-1/1	0.	0.
2:00	0.	0.
3:00	0.	0.
4:00	0.	0.
5:00	0.	0.
6:00	0.	0.
7:00	0.	0.
8:00	0.	0.
9:00	0.	0.
10:00	0.	0.
11:00	0.	0.
12:00	0.	0.
13:00	0.	-.04
14:00	0.	-.07
15:00	0.	-.07
16:00	0.	-.07
17:00	+.04	-.11
18:00	+.04	-.14
19:00	+.11	-.14
20:00	+.14	-.14
21:00	+.18	-.11
22:00	+.22	-.07
23:00	+.25	-.04
0:00-1/2	0.	0.
1:00	0.	0.

TABLE 6

Test for Updating of Besselian Constants,

1st Differences Only

Time GMT	Azimuth Diff. arcseconds	Elevation Diff. arcseconds
1:00-1/1	0.	0.
2:00	0.	0.
3:00	0.	0.
4:00	0.	0.
5:00	0.	0.
6:00	0.	0.
7:00	0.	0.
8:00	0.	0.
9:00	0.	+.04
10:00	0.	0.
11:00	0.	0.
12:00	0.	-.04
13:00	0.	-.04
14:00	0.	0.
15:00	0.	0.
16:00	0.	0.
17:00	0.	0.
18:00	0.	0.
19:00	0.	0.
20:00	0.	0.
21:00	0.	0.
22:00	0.	0.
23:00	0.	0.
0:00-1/2	0.	0.
1:00	0.	0.

From Table 5 one can observe that the differences increase with time. Also, in the original runs the differences decreased with smaller elevation angle. Note that these differences are much larger than those caused by omitting the second order interpolation. The differences caused by use of second order interpolation as seen in Table 6 are small. These differences will rarely be significant. See the graph of second order coefficient values in the next section.

4.4 Small Effects

A. Linear Change to Equation of Equinoxes

The parameter known as Equation of Equinoxes is the difference between the mean longitude of the Vernal Equinox at 0:00 hour, UT1, and the "True" longitude (i.e., inclusion of periodic terms). This difference is on the order of 0.8 seconds and changes by about .005 seconds per day. Since the updated Besselian constants transform the coordinates to the instantaneous "true" equinox, the Earth spin must be referenced to that same equinox. This is achieved by the following equations:

EOFE2 = EOFE + DEFE * XL11

ELNG = ELNGO + EOFE2

where

ELNG = "True" longitude of the Vernal Equinox,

ELNGO = "Mean" longitude of the Vernal Equinox,

EOFE = "True" - "Mean" longitude at the same epoch,

DEFE = first difference of EOFE (per day),

XL11 = fraction of elapsed day since 0:00 hour (UT1 time), and

EOFE2 = updated Equation of Equinoxes.

The differences between Equation of Equinox "Corrected" minus "Standard" (no corrections) as reflected in the look angles are listed in Table 7. A hand check of the 2nd differences for the Equation of Equinoxes showed that their size did not warrant their use in this program.

TABLE 7

Test for Linear Update to Equation of Equinoxes

Time GMT	Azimuth Diff. arcseconds	Elevation Diff. arcseconds
1:00-1/1	0.	0.
2:00	0.	0.
3:00	+.04	0.
4:00	+.04	0.
5:00	-.04	+.04
6:00	-.04	0.
7:00	-.04	0.
8:00	-.07	0.
9:00	-.04	0.
10:00	-.04	-.04
11:00	0.	-.04
12:00	0.	0.
13:00	0.	-.04
14:00	0.	-.04
15:00	+.04	-.04
16:00	+.04	-.04
17:00	+.04	-.04
18:00	+.04	-.04
19:00	+.04	0.
20:00	+.07	0.
21:00	+.07	0.
22:00	+.04	+.04
23:00	+.04	+.04
0:00-1/2	0.	0.
1:00	0.	0.

B. Earth Spin Rate Changes

The second small change is in the Earth's spin rates. The constant part, WE, is the value for epoch 1900. There is a secular increase, DW, whose value is:

T = time in days since 1900

TT = T/36525

DW = 4.29 D-15 * TT in radians/second

Then the corrected spin rate (relative to the moving Vernal Equinox) is:

WED1 = WE + DW

Here, $WE = 7.29211585468 * 10^{-5}$ radians/second, and

$DW \simeq 3.25 * 10^{-14}$ for 1976.

However, there is another correction due to polar motion. This is a scale factor of the form:

WED = WED1 * (1. + DUTDOT)

where DUTDOT is the rate of change of UT1 in the dimensionless form of days/day. DUTDOT is preset to -2.0 milliseconds/day (unless changed by polar motion corrections). Hence,

$DUTDOT = -2.0$ milliseconds/day $= -2.3 * 10^{-8}$ days/day, and

1. + DUTDOT = .999999977.

In particular, $WED - WED1 \simeq -2.79 * 10^{-11}$ radians/second. Hence, the principal effort is that of the motion of the spin axis (polar motion rate) at the present time. The secular change is, or course, progressive.

The effect of this on look angles amounts to no more than 0.04 arcseconds during any day. See Table 8 for a list of these differences ("corrected" - "uncorrected", as usual).

TABLE 8
Test for Correction to Earth Spin Rate

Time GMT	Az. Diff. arcsec.	El. Diff. arcsec.	Time GMT	Az. Diff. arcsec.	El. Diff. arcsec.
1:00-1/1	0.	0.	13:00	0.	0.
2:00	0.	0.	14:00	-.04	+.04
3:00	0.	0.	15:00	0.	+.04
4:00	0.	0.	16:00	0.	0.
5:00	0.	0.	17:00	0.	0.
6:00	0.	0.	18:00	0.	0.
7:00	+.04	0.	19:00	-.04	0.
8:00	+.04	0.	20:00	0.	0.
9:00	+.04	0.	21:00	0.	0.
10:00	+.04	0.	22:00	-.04	0.
11:00	+.04	+.04	23:00	-.04	-.04
12:00	0.	0.	0:00-1/2	0.	0.
			1:00	0.	0.

C. Second Order Effects

The second order effects are the cross-terms between aberration corrections performed in a fixed coordinate system and nutation effects, which occur in a moving coordinate system. These effects are very small except for stars at high declination angles. In the case considered above the effect on azimuth is, at most, .04 arcseconds for, on the average, one point per day. However, in Table 9 below, based on a Right Ascension of 21 hours for day 26, 1976, it is shown how this effect grows. For stars with declination below 60° absolute value, second order corrections need not be applied.

TABLE 9

Table of 2nd Order Effects Versus Star Declination

Declination degrees	Rt. Asc. Correction arcseconds	Dec. Correction arcseconds
0	0.	0.
10	-.00004	-.00017
20	-.00018	-.00035
30	-.00044	-.00056
40	-.00093	-.00081
50	-.00188	-.00115
60	-.00396	-.00167
70	-.00997	-.00264
80	-.04250	-.00546
89	-4.33669	-.05513
-10	+.00002	+.00016
-20	+.00006	+.00032
-30	+.00016	+.00051
-40	+.00035	+.00074
-50	+.00070	+.00105
-60	+.00147	+.00153
-70	+.00370	+.00243
-80	+.01576	+.00501
-89	+1.60842	+.05061

D. Total Small Efforts

Table 10 below portrays the differences between the uncorrected data and the data with all three small effects corrected. Again, the differences are "corrected" - "uncorrected".

TABLE 10

Test for Total Small Errors Effects

Time GMT	Azimuth Diff. arcseconds	Elevation Diff. arcseconds
1:00-1/1	0.	0.
2:00	0.	0.
3:00	+.04	0.
4:00	+.04	0.
5:00	0.	+.04
6:00	0.	0.
7:00	0.	0.
8:00	-.04	0.
9:00	-.04	0.
10:00	0.	0.
11:00	0.	-.04
12:00	0.	0.
13:00	0.	0.
14:00	0.	0.
15:00	0.	0.
16:00	+.04	0.
17:00	+.04	-.04
18:00	+.04	0.
19:00	0.	0.
20:00	+.04	0.
21:00	+.04	0.
22:00	0.	+.04
23:00	+.04	0.
0:00-1/2	0.	0.
1:00	0.	0.

4.5 Effect of Proper Motion and Parallax

A. Parallax

This correction is, in general, small. In fact, it can almost always be neglected if unknown. The known values on parallax can be found in the Yale University Catalog and its supplements. A copy is in the possession of Dr. George Sinclair at FEC/Performance Analysis Department.

The largest parallax known has a value of 0.762 arcseconds. However, only 66 stars have parallaxes greater than or equal to 0.100 arcseconds. Further, the largest negative parallax is -.057 arcseconds. In Table 11, runs with these two parallaxes are compared against a run with no parallax. Again, "difference" = "corrected" - "uncorrected".

One should note that, in the expression for parallax correction (written below in rectangular coordinates), it is assumed that the Earth's orbit is circular and unperturbed and that the Besselian numbers C and D are referenced to the Sun's center. *Actually, the Besselian numbers C and D are referenced to the* barycenter of the solar system (approximately). However, these corrections, when multiplied by the parallax, are effectively of third order and can be neglected. The equations mentioned above are:

$$\Delta X = C*PAR/(CABER*DCOS(E))$$

$$\Delta Y = D*DCOS(E)*PAR/CABER$$

$$\Delta Z = \Delta Y*DTAN(E)$$

where,

CABER = 20.496 arcseconds = solar aberration constant,

E = mean obliquity of Earth, referenced to nearest Besselian New Year,

PAR = parallax, and

C, D = Besselian constants for this day.

TABLE 11

Effect of Parallax Corrections

	Parallax = +.752		Parallax = -.057		Parallax = +.1	
Time GMT	Az. Diff. arcsecond	El. Diff. arcsecond	Az. Diff. arcsecond	El. Diff. arcsecond	Az. Diff. arcsecond	El. Diff. arcsecond
1:00-1/1	-.36	+.25	+.04	0.	-.04	+.04
2:00	-.43	+.18	+.04	0.	-.07	+.04
3:00	-.50	+.11	+.07	0.	-.04	0.
4:00	-.58	0.	+.07	0.	-.07	0.
5:00	-.72	-.04	+.04	+.04	-.11	0.
6:00	-.79	-.18	+.04	0.	-.11	-.04
7:00	-.72	-.30	+.07	+.04	-.11	-.04
8:00	-.14	-.43	0.	0.	-.04	-.07
9:00	+.58	-.36	-.04	+.04	+.07	-.04
10:00	+.79	-.25	-.04	0.	+.11	-.04
11:00	+.72	-.14	-.04	0.	+.11	-.04
12:00	+.61	0.	-.07	0.	+.07	0.
13:00	+.54	+.07	-.04	0.	+.07	0.
14:00	+.43	+.14	-.04	0.	+.04	+.04
15:00	+.40	+.22	-.04	0.	+.04	+.04
16:00	+.30	+.29	-.04	-.04	+.04	+.04
17:00	+.22	+.30	0.	-.04	+.04	+.04
18:00	+.18	+.36	0.	-.04	+.04	+.04
19:00	+.11	+.40	-.04	-.04	0.	+.04
20:00	+.04	+.43	0.	-.04	0.	+.07
21:00	-.07	+.43	0.	-.04	0.	+.07
22:00	-.18	+.40	0.	0.	-.04	+.07
23:00	-.25	+.36	0.	-.04	-.04	+.04
0:00-1/2	-.29	+.29	+.04	-.04	-.04	+.04
1:00	-.36	+.25	+.04	0.	-.04	+.04

Note above that the corrections are larger than the design accuracy of 0.01 arcseconds. Hence, if the parallax is known, it should be used in STAR1 runs.

B. Effect of Proper Motion

Proper motion changes the mean coordinates of the star over a year's period. Hence, there will be no effect from proper motion at the beginning and end of the year, and there will be a maximum effect at July 1 (mid-year). Note, here, that the reference epoch is changed at July 1 so that the size of the annual effects are minimized. Table 12 lists the differences ("run with proper motion" - "std".) for various combinations of proper motions, all large, on day 182. So, this table can be considered a table of "worst" cases. UR and UD are listed in arcseconds/tropical year.

Note the size of the differences. In order to preserve the accuracy standards of this program, proper motion corrections must be included if either UR or UD is larger than 0.1 arcseconds/tropical year <u>and</u> if the month of the run time is between March and October.

TABLE 12

Effect of Large Proper Motion Corrections

UD =	+2.00		0.		+2.00		-2.00		0.		-2.00	
UR =	0.		+3.606		+3.606		0.		-3.61		-3.61	
Time	AZ arcsecond	EL	AZ arcsecond	EL	AZ arcsecond	EL	AZ arcsecond	EL	AZ arcsecond	EL	AZ arcsecond	EL
1:00-182	+1.30	+0.14	-0.22	+0.97	+1.08	+1.12	-1.30	-0.18	+0.22	-0.97	+1.08	-1.15
4:00	+0.79	+0.68	-0.68	+0.77	+0.11	+1.40	-0.77	-0.60	+0.68	-0.72	-0.07	-1.37
7:00	+0.29	+0.97	-0.94	+0.29	-0.68	+1.22	-0.25	-0.94	+0.97	-0.25	+0.72	-1.19
10:00	-0.36	+0.94	-0.94	-0.30	-1.30	+0.58	+0.36	-0.94	+0.94	+0.36	+1.26	-0.58
13:00	-0.86	+0.58	-0.60	-0.83	-1.48	-0.22	+0.86	-0.61	+0.61	+0.79	+1.48	+0.22
16:00	-1.40	+0.07	+0.07	-0.97	-1.48	-0.94	+1.37	-0.07	+0.07	+0.97	+1.48	+0.94
19:00	-1.69	-0.77	+1.94	-0.60	+0.25	-1.40	+1.66	+0.77	-1.98	+0.60	-0.29	+1.37
22:00	+1.84	-0.58	+1.30	+0.79	+3.13	+0.25	+1.51	+0.58	-1.33	-0.79	-3.17	-0.25
23:30	+1.62	-0.18	+0.29	+0.97	+1.87	+0.83	-1.58	+0.14	+0.22	-0.97	-1.84	-0.83

4.6 Effect of UT1 Correction

A. Values of Polar Motion Coefficients

The polar motion phenomena can be represented by three numbers. The first is the difference in time = "UT1" - "UTC" (in seconds). The last two refer to the offset of the pole from the CIO (Conventional International Origin) along the X and Y axis of the tangent plane. These offsets are in arcseconds. Sets of these numbers are available in the "US Naval Observatory Bulletin" (Series 7), "BIH Circular D119", and "Monthly Notes of the International Polar Motion Service." Preliminary estimates (before epoch) are:

1. DUT1 correction. (Time only); accurate to +.05 seconds (with respect to other preliminary estimates). This is changed monthly.
2. Estimate correction. This is a time equation, good for 60 days and available from the Series 7 bulletin.
3. Extrapolated correction. Time only; this is a daily estimate for a week, and is available one week ahead. The rate of change of UT1 - UTC (in milliseconds/day) is also available in this table of estimates.

Measured values available are:

1. USNO Rapid Service. Daily values of UT1 - UTC, X, and Y for a week, available one week after the epoch. This is obtained from satellite tracking.
2. BIH (Bureau International de l'Heure) values. Values of UT1 - UTC, X, and Y are available for every 5 days covering a month. These values are both raw and smoothed. They are available one month late. These values are obtained from both satellite tracking and astronomic latitude data.
3. DPMS (Doppler Polar Monitoring Service). Values of X and Y, available every 5 days, approximately two weeks late. These values are obtained from doppler tracking of satellites. Use the Rapid Service time (UT1 - UTC) or the BIH raw time with these values.

4. IPMS (International Polar Motion Service). Values of X and Y, available about every 18 days (every 0.05 year based on the Besselian year) These values are both final (about 1/3 year late) and preliminary (about 1/4 year late). The values are based on a multistation solution using observations from about 50 astronomical latitude observatories.

5. ILS (International Latitude Service). Values of X and Y, available for every 0.05 part of the Besselian year. These values are about 1/3 year late. They are a weighted average of the values from 6 specific astronomical latitude observatories. The BIH smooth times should be used here and with source 4 above.

For comparison purposes, Tables 13, 14, and 15 show the various values at 4 times during 1976. All values in Tables 13 and 14 are "UT1" - "UTC".

TABLE 13

Preliminary Times for Use in Polar Motion Tests

Date	Extrapolated seconds	Estimated seconds	DUT1 seconds	DUTDOT* milliseconds/day
2/1	+.626	+.636	+.6	-2.5
4/1	+.470	+.454	+.4	-3.3
7/1	+.182	+.179	+.2	-2.7
10/1	-.044	-.047	-.1	-3.0

* DUTDOT is the rate of change of "UT1 - UTC"

TABLE 14

Final Times for Use in Polar Motion Tests

Date	Rapid Service seconds	BIH - Raw seconds	BIH - Smooth seconds
2/1	+.6417	+.6399	+.6393
4/1	+.4587	+.4575	+.4570
7/1	+.1862	+.1880	+.1865
10/1	-.0510	-.0518	-.0509

TABLE 15

Polar Coordinate Corrections from Available Sources†

Date	Rapid Service		BIH - Raw		BIH - Smooth		DPMS		IPMS††		ILS	
	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y
2/1	-.138	+.239	-.128	+.267	-.138	+.274	-.158	+.278			+.087	+.260
4/1	-.075	+.392	-.106	+.402	-.100	+.399	-.135	+.406			-.002	+.416
7/1	+.126	+.380	+.141	+.385	+.136	+.390	+.150	+.398			+.094	+.348
10/1	+.271	+.155	+.223	+.150	+.243	+.158	+.187	+.154	+.214	+.170	+.145	+.144

Note that, for dates not occurring on the epoch time of the data values, a correction was made via linear interpolation.

† All coordinates (X and Y) are in arcseconds.

†† The IPMS reports started on 2/3/1977. So, no comparison with preliminary values is available.

B. Effect on Look Angles

The following comparison runs were made.

Time Only	Time, X, Y
1. Extrapolated	8. Rapid Service Values
2. Estimated	9. BIH Smoothed Values
3. DUT1	10. BIH Raw Values
4. Rapid Service Time	11. DPMS, BIH (Raw) Time Values
5. BIH (Raw) Time	12. DPMS, BIH (Smoothed) Time Values
6. BIH (Smoothed) Time	13. ILS, BIH (Smoothed) Time Values
7. No Correction	14. IPMS, BIH (Smoothed) Time Values

The first and most important comparison will be between the uncorrected or "std" run and both Rapid Service runs (with and without X, Y, values). The DUT1 corrected run is also compared. This comparison is shown in Table 16 in the sense "Rapid Service" - "Std." and "DUT1" - "Std.".

TABLE 16

Comparison of Polar Motion Corrected Data with Uncorrected Data

	RS (Time only)		RS (Time, X, Y)		DUT1	
	AZ	EL	AZ	EL	AZ	EL
Time	arcseconds		arcseconds		arcseconds	
1:00-1/1	-.25	-.30	-.60	-.30	-.50	-.68
4:00	0.	-.40	-.40	-.40	-.04	-.83
7:00	+.83	-.25	+.43	-.25	+1.73	-.50
10:00	+.50	-.30	+.11	-.30	+1.04	+.04
13:00	-.11	+.40	-.50	+.40	-.22	+.83
16:00	-.29	+.29	-.68	+.29	-.56	+.61
19:00	-.40	+.11	-.79	+.11	-.83	+.18
22:00	-.40	-.14	-.79	-.14	-.79	-.29
23:30	-.30	-.25	-.72	-.25	-.68	-.50

As can be seen, the DUT1 correction may overcorrect as much as 1.5 arcseconds. In runs such as these the Estimated correction is the best long-term correction available.

Let us compare the RS (Time Only) run against the other time runs (1, 2, 5, and 6). There will be little difference in the results, tabulated in Table 17 in the sense "Run" - "RS". The results of the other runs with X, Y data (numbers 9 to 14) will also be compared with the "RS with X, Y data" run. This comparison is listed in Table 18 in the same sense ("Run" - "RS"). Again, there is little difference in the effect of the different data. Finally, the data from runs with X, Y data will be differenced with data from runs with only time corrections (the same as used for the X, Y run). These differences are listed in Table 19 in the sense "Run with X, Y" - "Run with time only". These differences are in the azimuth channel only. Note that the difference simulates a simple bias. The input values for time errors for the various runs are listed below.

Values Used

DUTDOT = -3.7 milliseconds/day

Type	Extrap	Estim.	DUT1	Rpd. Srv.	BIH Raw	BIH Smooth	DPMS	IPMS	ILS
Time (seconds)	-.041	-.04037	-.1	-.0476	-.04876	-.04766	—	—	—
X (arcseconds)	-	-	-	+.272	+.217	+.2448	+.1408	+.215	+.145
Y (arcseconds)	-	-	-	+.161	+.1526	+.1606	+.1538	+.172	+.145

TABLE 17

Comparison of the Effects of Time Estimates of Polar Motion

Date UTC	Extrap Time AZ arcseconds	Extrap Time EL arcseconds	Estimated Time AZ arcseconds	Estimated Time EL arcseconds	BIH Time (Raw) AZ arcseconds	BIH Time (Raw) EL arcseconds	BIH Time (Smooth) AZ arcseconds	BIH Time (Smooth) EL arcseconds	DUT1 AZ arcseconds	DUT1 EL arcseconds
1:00-1/1	-.04	+.04	+.04	+.07	0.	0.	0.	0.	-.25	-.36
4:00	0.	+.04	0.	+.04	0.	0.	0.	0.	-.04	-.43
7:00	-.11	+.04	-.14	+.04	0.	0.	0.	0.	+.90	-.25
10:00	-.07	-.04	-.07	-.04	0.	0.	0.	0.	+.54	+.36
13:00	+.04	-.07	+.04	-.07	0.	0.	0.	0.	-.11	+.43
16:00	+.04	-.04	+.07	-.04	0.	0.	0.	0.	-.29	+.30
19:00	+.04	-.04	+.07	-.04	0.	0.	0.	0.	-.43	+.07
22:00	+.07	+.04	+.07	+.04	0.	0.	0.	0.	-.40	-.14
23:30	+.04	+.04	+.04	+.07	0.	0.	0.	0.	-.36	-.25

Only the DUT1 correction produced large differences from the Rapid Service post-epoch data. For precision work the DUT1 correction can not be used. Note also, from Table 16, that failure to correct polar motion causes relatively large errors.

TABLE 18

Comparison of the Effects of Time and Polar Coordinate Estimates of Polar Motion

	BIH (raw)		BIH (sm.)		DPMS (BIH raw t)		DPMS (BIH sm. t)		IPMS (BIH sm. t)		ILS (BIH sm. t)	
	AZ	EL	AZ	EL	AZ	EL	AZ	EL	AZ	EL	AZ	EL
	arcseconds		arcseconds		arcseconds		arcseconds		arcseconds		arcseconds	
1:00-1/1	+.07	0.	+.04	0.	+.11	0.	+.11	0.	+.07	0.	+.14	0.
7:00	+.07	0.	+.04	0.	+.11	0.	+.07	0.	+.04	0.	+.14	0.
13:00	+.07	0.	+.04	0.	+.11	0.	+.11	0.	+.07	0.	+.14	0.
19:00	+.04	0.	+.04	0.	+.07	0.	+.07	0.	+.04	0.	+.14	0.
23:30	+.07	0.	+.04	0.	+.07	0.	+.11	0.	+.04	0.	+.14	0.

The largest differences in Table 18 occur between RS and ILS data, as might be expected. Since there is no difference in elevation, only the differences in azimuth are listed in Table 19.

TABLE 19

Comparison of Data Corrected for Polar Motion Time and Pole Coordinates with Data Corrected for Polar Motion Time Only

Name	Rapid Service	BIH (raw)	BIH (smooth)	DPMS (raw)	DPMS (smooth)	IPMS	ILS
Run #	'8'-'1'	'10'-'5'	'9'-'6'	'11'-'5'	'12'-'6'	'14'-'6'	'13'-'6'
Time	AZ arcseconds	AZ arcseconds	AZ arcseconds	AZ arcseconds	AZ arcseconds	AZ arcseconds	AZ arcseconds
1:00-1/1	-.40	-.30	-.36	-.29	-.29	-.30	-.26
4:00	-.40	-.30	-.36	-.25	-.30	-.36	-.26
7:00	-.40	-.30	-.36	-.29	-.29	-.36	-.26
10:00	-.40	-.30	-.36	-.22	-.30	-.36	-.26
13:00	-.40	-.30	-.36	-.29	-.29	-.30	-.26
16:00	-.40	-.30	-.36	-.22	-.29	-.30	-.26
19:00	-.40	-.36	-.36	-.30	-.29	-.36	-.26
22:00	-.40	-.30	-.36	-.22	-.30	-.30	-.26
23:30	-.40	-.30	-.36	-.30	-.29	-.36	-.26

The average differences above is -1/3 arcseconds. This can be thought of as a general penalty for failure to use the X, Y coordinates.

C. Comments

In view of Table 16, DUT1 should not be used in this program. For best results use the Extrapolated values for the day in question. The Estimated value (good for 60 days in advance) will also produce good results.

For post-epoch work the Rapid Service values are as good as any. However, in view of the differences shown in Table 14 for different epochs, the Rapid Service values must be checked for the occurrence of large differences between these values and the refined BIH raw values. The smoothed BIH values might be better for some applications such as long-term studies.

The various values of X, Y available, with the exception of the ILS values (which should be used only with studies requiring data available before 1965), are almost equal. In particular, the difference between any two modulii ($= \sqrt{(x^2 + y^2)}$) is on the order of .05 arcseconds. Due to the interpolation required, the IPMS values are less accurate than the BIH, or DPMS, or Rapid Service data near the middle of an interpolation period.

4.7 Refraction Correction

Provision has been made in STAR1 to accommodate different refraction subroutines. However, only a simple algorithm (Refrac Method #1) has been implemented. In Table 20, a typical example is shown. As usual, Delta = EL (refracted) - EL(true). EL(true) is also listed. There is no azimuth correction in this model. The refraction equation† is given below, followed by Table 20.

EL(refracted) = EL(true) + 2.73D-4*RAD/DTAN(EL(true))

The above equation is good for <u>optical</u> refraction. For RF refraction, replace "2.73" with "3.36".

† The experience at WTR with this equation indicates it is surprisingly accurate at elevations above 15°. The two numerical values can be computed from formulas (for $N \times 10^{-6}$) on pages 2 and 3 of: Landry, P.M. and Parks, L.D.; "Atmospheric Refraction Effects on Tracking System Data"; APGC Tech. Rpt. #APGC-TDR-63-28 (1963). Input values are: T = 62°F, e = 15 mb, P = 1008 mb (effective height of 250 meters).

TABLE 20

Comparison of Refraction Corrected Data with Uncorrected Data

Time	EL (True) degrees	Delta arcminutes,arcseconds
1:00-1/1	41.11157	+1' 4.53"
2:00	33.04827	1' 25.09"
3:00	25.50169	1' 58.06"
4:00	18.72165	2' 46.14"
5:00	12.95303	4' 4.82"
6:00	8.43667	6' 19.63"
7:00	5.39193	9' 56.61"
8:00	3.98658	13' 28.00"
9:00	4.30438	12' 28.13"
10:00	6.32602	8' 27.93"
11:00	9.93365	5' 21.55"
12:00	14.93664	3' 31.07"
13:00	21.10474	2' 25.87"
14:00	28.19450	1' 45.04"
15:00	35.96043	1' 17.60"
16:00	44.14615	58.03"
17:00	52.44546	43.32"
18:00	60.39787	31.37"
19:00	67.12341	23.75"
20:00	70.87796	19.80"
21:00	69.76861	20.74"
22:00	64.45318	26.91"
23:00	57.04342	36.51"

4.8 Change in Apparent Position

The final result treated in this section is the change in apparent star position over a short time period. Table 21 below has a comparison of apparent star positions of the same star over 1/2 hour intervals. It is clear from this table that, for high accuracy, the apparent position must be updated to the time of look angle computation rather than daily.

TABLE 21
Effect of Apparent Position Updating

Time	Declination			Right Ascension		
	Degrees	Min	Seconds	Hours	Min	Seconds
1:00	-8	13	21.9350	+5	13	23.1345
1:30	-8	13	21.9330	+5	13	23.1342
2:00	-8	13	21.9310	+5	13	23.1338
2:30	-8	13	21.9291	+5	13	23.1335
3:00	-8	13	21.9271	+5	13	23.1331
3:30	-8	13	21.9252	+5	13	23.1328

Declination (1:00) - Declination (3:30) = 0.0098 arcseconds

RT. Ascension (1:00) - RT. Ascension (3:30) = 0.0017 seconds = 0.0255 arcseconds

VERIFICATION OF SUBROUTINES

5.0 INTRODUCTION

The subroutines used in STAR1 were verified by direct calculation and comparison for the simple subroutines MV, TIMEE, OCT, OCTO, and ASDCRK. Simple debug runs of both TIMEE and MV are listed below. ASDCRK, OCT, and OCTO require special consideration. The remaining routines were previously verified (VERNAL), required no verification (BLOCK DATA), or (for CNSTNT or CNSTJ) will be verified by comparison with data from reference 6 (hereafter called AENA).

5.1 Verification of MV and TIMEE

MV multiples the matrices Q and V to obtain the matrix O. The matrix Q is a 3x3 matrix and V and O are 3x1 vectors. In this case,

$$V = \begin{bmatrix} 0.19995884687 \\ 0.96921698587 \\ -0.14315545789 \end{bmatrix} \text{ and}$$

$$Q = \begin{bmatrix} 1. & -1.1271198623 \times 10^{-4} & -4.8937153183 \times 10^{-5} \\ 1.1271198623 \times 10^{-4} & 1. & 2.9917810259 \times 10^{-5} \\ 4.8937153183 \times 10^{-5} & -2.9917810259 \times 10^{-5} & 1. \end{bmatrix}$$

The results are:

	MV	Calc.	Diff.
0(1) =	0.1998566106	0.1998566101	-5×10^{-10}
0(2) =	0.9692352415	0.9692352408	-7×10^{-10}
0(3) =	-0.143176693	-0.1431746693	0.

where DIFF = Calc. - MV. The difference is due to roundoff and lack of precision in the calculated portion. The MV column above is taken from the computer printout.

We will similarly verify TIMEE. The TIMEE column is taken from a computer printout. Below, the difference between GMT and local time is +8 hours.

Variable	TIMEE	Calc.
LD	2	2
LS	3600	3600
IDG	2	2
GH	1.0	3600/3600 = 1.0
IHG	1	1
MG	0.	1.0 - 1 = 0.
IDL	2	2
XLH	-7	1. - 8 = -7.
IDL	1	2 - 1 = 1
XLH	17.0	-7. + 24. = 17.0
IHL	17	17
ML	0.	17.0 - 17 = 0.

Hence, TIMEE and MV are verified.

5.2 Verification of ASDCRK

The ASDCRK subroutine contains both of the transformations X, Y, Z cosines to Right Ascension - Declination coordinates; that is, (X, Y, Z) to (RAS, DEC) and the converse (RAS, DEC) to (X, Y, Z). Hence, the double transformation (RAS, DEC) to (X, Y, Z) and (X, Y, Z) to (RAS, DEC) was repeated 20 times for example verification. The final values of (RAS, DEC) were then compared with the initial values. The results were:

$$RAS(f) - RAS(i) < 5. \times 10^{-17} \text{ radians}$$

$$DEC(f) - DEC(i) = -1. \times 10^{-16} \text{ radians} = -2.05 \times 10^{-12} \text{ arcseconds}$$

The initial and final X, Y, Z components could also be compared. These results were:

$$X(f) - X(i) < 5. \times 10^{-17} \text{ radians}$$

$$Y(f) - Y(i) < 5. \times 10^{-17} \text{ radians}$$

$$Z(f) - Z(i) < 5. \times 10^{-17} \text{ radians}$$

Further,

$$|X^2 + Y^2 + Z^2 - 1.| < 2. \times 10^{-11}$$

One can conclude that one part of the transformation is the inverse of the other part. So, all that one must do is verify one side of this subroutine. The side chosen is the transformation from (X, Y, Z) to (RAS, DEC), since this involves the arctrig functions.

Initially, the transformation was written:

```
DEC = DARSIN(X(3))
```

where

X(1) = X, X(2) = Y, and X(3) = Z.

This result was compared with the 12 - place trig subroutines on the WANG. The results differed by as much as 20 arcseconds.

By referring to the IBM manual of system subroutines, it was discovered that the DATAN subroutine was more accurate in the small angle region. Jerry Trimble of FEC/Performance Analysis Department provided the author unpublished results of a numerical comparison of the IBM DATAN and DARSIN subroutines. DATAN was as much as 10 times more accurate for small angles. Hence, the transformation was changed to:

```
DEL = DSQRT(X(2)*X(2) + X(1)*X(1))
DEC = DATAN(X(3)/DEL)
```

The other coordinate transformation is:

```
RAS = DATAN2(X(2),X(1))
```

These transformations were checked with many values of (X, Y, Z) to prove that the resulting angles were in the correct quadrant. The numerical values

were compared with the WANG 12-place trig pack values. The results are listed below:

$$\text{DEL(STAR1)} - \text{DEL(WANG)} < 4 \times 10^{-12} = 8 \times 10^{-7} \text{ arcseconds,}$$

$$\text{DEC(STAR1)} - \text{DEC(WANG)} < 3 \times 10^{-12} \text{ radians} = 6 \times 10^{-7} \text{ arcseconds, and}$$

$$\text{RAS(STAR1)} - \text{RAS(WANG)} < 9 \times 10^{-12} \text{ radians} = 2 \times 10^{-6} \text{ arcseconds.}$$

These results are at the limit of the WANG's accuracy.

Selected values were then computed by hand using the tables available in reference 11 (see page 92 and Table 4.14). The maximum difference was less than 0.0005 arcseconds between values computed from the tables and values computed via ASDCRK. Still, this subroutine will fail for declinations of 90° and -90°. Recall that this program presupposes that no star has such declination. The Right Ascension Calculation has no such problem.

5.3 Verification of OCT and OCTO

These two subroutines can be verified simultaneously by comparing octal output against angle input for several encoder bit values. This is true because OCTO sets the parameters for OCT. The only part of OCT not verified in this manner is the round-up loop. Examination of the example in Section 3 will provide verification of this part of the program except for the values of LIP provided by OCTO. A simple hand calculation was sufficient to verify these three values.

The verification of the central portion of OCT (and OCTO) proceded in two steps. First, the octal number was changed to base 10; second, this number was multiplied by the LSB (least significant bit size) of that encoder. The resulting angle was then compared with the input angle - specifically, DELTA = Input Angle - Calculated Angle. These results are summarized in Table 23. A list of LSB versus encoder bit size can be found in Table 22.

Part 1

TABLE 22

TABLE OF ANGLE LSB VALUES

DEGREES	BIT	MILS
180	1	3200.
90	2	1600.
45	3	800.
22.5	4	400.
11.25	5	200.
5.625	6	100.
2.8125	7	50.
1.40625	8	25.
0.703125	9	12.5
0.3515625	10	6.25
0.17578125	11	3.125
0.087890625	12	1.5625
0.0439453125	13	.78125
0.02197265025	14	.390625
0.010986328125	15	.1953125
0.0054931640625	16	.09765625
0.00274658203125	17	.048828125
0.001373291015625	18	.0244140625
0.0006866455078125	19	.01220703125
0.00034332275390625	20	.006103515625
0.000171661376953125	21	.0030517578125
0.0000858306884765625	22	.00152587890625
0.00004291534423828125	23	.000762939453125
0.000021457672119140625	24	.0003814697265625
0.0000107288360595703125	25	.00019073486328125
0.00000536441802978515625	26	.000095367431640625
0.000002682209014892578125	27	.0000476837158203125
0.0000013411045074462890625	28	.00002384185791015625
0.00000067055225372314453125	29	.000011920928955078125
0.000000335276126861572265625	30	.0000059604644775390625
0.0000001676380634307861328125	31	.00000298023223876953125
0.00000008381903171539306640625	32	.000001490116119384765625
0.000000041909515857696533203125	33	.0000007450580596923828125
359 99999995[illegible]09048414230346679 6875	TOTAL	6399.9999992549419403076171875

ARCSECONDS	BIT	MILLIRADIANS
648000.	1	3141.592653589793238462644
324000.	2	1570.796326794896619231322
162000.	3	785.398163397448309615661
81000.	4	392.699081698724154807830
40500.	5	196.349540849362077403915
20250.	6	98.174770424681038701958
10125.	7	49.087385212340519350979
5062.5	8	24.543692606170259675490
2531.25	9	12.271846303085129837745
1265.625	10	6.135923151542564918872
632.8125	11	3.067961575771282459436
316.40625	12	1.533980787885641229718
158.203125	13	.766990393942820614859
79.1015625	14	.383495196971410307430
39.55078125	15	.191747598485705153715
19.775390625	16	.095873799242852576858
9.8876953125	17	.047936899621426288429
4.94384765625	18	.023968449810713144214
2.471923828125	19	.011984224905356572107
1.2359619140625	20	.005992112452678286054
.61798095703125	21	.002996056226339143027
.308990478515625	22	.001498028113169571514
.1544952392578125	23	.000749014056584785757
.07724761962890625	24	.000374507028292392878
.038623809814453125	25	.000187253514146196439
.0193129049072265625	26	.000093626757073098220
.00965645245361328125	27	.000046813378536549110
.004828226226806640625	28	.000023406689268274555
.0024141113113403320312	29	.000011703344634137278
.0012070556556701660156	30	.000005851672317068640
.0006035278278350830078	31	.000002925836158534320
.0003017641391754150339	32	.000001462918079267160
.0001508820695877075520	33	.000000731459039633580
1295999.9998511179304122929800		6283.185306448127437291754

TABLE 23

Comparisons of OCT Output with Input

BIT Size	LSB deg	INPUT deg	OUTPUT octal	OUTPUT base 10	OUTPUT x LSB deg	DELTA ARS†	DELTA lsb††
13	$4.39453125 \times 10^{-2}$	333.01368	16632	7578	333.01758	-14.03	-.09
17	$2.74658203 \times 10^{-3}$	331.03715	353317	120527	331.03729	-0.51	-.05
27	$2.68220901 \times 10^{-6}$	331.03729	726636000	123419648	331.03729	-.007	-.68
13	$4.39453125 \times 10^{-2}$	5.57545	177	127	5.58106	-20.18	-.13
17	$2.74658203 \times 10^{-3}$	5.68306	4025	2069	5.68268	+1.37	+.14
27	$2.68220901 \times 10^{-6}$	1.53020	2132203	570499	1.53020	+.009	+.91

† ARS = arcsecond

†† lsb = fraction of the LSB value

All results are within an LSB of the input with respect to the particular encoder bit size.

5.4 Verification of CNSTNT and CNSTJ

A. CNSTJ ENTRY

The subroutine CNSTJ was verified by using it to compute its output values, J and J', at ten day intervals throughout 1976 and comparing the output against AENA (pages 328 to 331). In 1924 entries there were only 28 errors - of a maximum value of 1 unit (In J the units are 1×10^{-5} seconds; in J' they are 1×10^{-4} arcseconds). The errors were equally divided between the northern and soutnern hemispheres. The average error (sum of errors divided by 28) is -1.8×10^{-5} arcseconds or approximately -0.2 units. Hence, the errors are thought to be due to rounding only.

B. Constants Computed in CNSTNT

The subroutine CNSTNT required more involved verification. The constants computed in CNSTNT are E (Earth's obliquity), EOFE and DEFE (difference between the "Apparent" and "Mean" Vernal Equinox and the first derivative of that difference), and DW (secular change in Earth's spin rate). Also, TTAU (fraction of a tropical year since the nearest Besselian New Year) is passed via Common. Finally, one must also compute DB (the UTC date of the Besselian New Year), DL (the length of that year), and XM (the annual precession in declination).

First, TTAU was compared against AENA (pages 308 to 322) for a full year. No difference was found for four decimal places (the accuracy of the numbers listed in AENA). DL and DW are single line computations of small errors. They were verified by hand calculation. The equations can be found in reference 2 (pages J10 and J8). Typical examples are:

$$DW = -3.26 \times 10^{-4} \text{ radians per second, and}$$

$$DL = 365.242198698 \text{ days, both for year 1976.}$$

The constant DB has already been partially verified, as it must be known in order to compute TTAU. However, a comparison table against values found in reference 4 (pages 434 - 435) is listed below.

TABLE 24

Besselian New Year Data Test

Year	Calculated Date	AENA Date
1900	Jan 0.81352	Jan 0.813
1964	Jan 1.3142075	Jan 1.314
1975	Jan 0.9783811	Jan 0.978
1976	Jan 1.2205786	Jan 1.221
1977	Jan 0.4627761	Jan 0.463

Two values of E are compared with AENA values below. The difference is due to lack of precision in the calculator used to change radians to degrees, minutes, and seconds.

E = 23°26'32.656" (AENA) or 23°26'32.655" (calc.) for year 1976

E = 23°26'32.187" (AENA) or 23°26'32.186" (calc.) for year 1977

Note that the equation for EOFE is: (AA(1) - XM)/ETN, where XM is the annual precession in declination. Also, XM is approximately 20 arcseconds/year and is the same numerical size as AA(1). Hence, if the computation for EOFE can be verified, the calculation of XM will also be verified. Since EOFE + DEFE = EOFE for the next day, one can easily check DEFE also.

Below is a comparison of calculated values of EOFE with those available in AENA (pages 12 - 19).

TABLE 25

Equation of Equinox Comparison, Year 1976

Date (day)	EOFE (calc.)	EOFE (AENA)
1	+.8428	+.843
2	.8499	.850
3	.8550	.855
4	.8579	.858
5	.8582	.858
99	.7131	.713
100	.7135	.713
101	.7102	.710
102	.7053	.705
180	.7078	.708
181	.7130	.713
182	.7164	.716
183	.7174	.717
360	.5642	.564
361	.5632	.563
362	.5608	.561

Note that the values agree to the nearest decimal that AENA lists. This completes the discussion of the constants computed.

C. First and Second Derivatives of Besselian Numbers

The assignment of Block Data values to the proper day, etc., was checked by hand against the printout. The derivatives are computed by using, for the first derivative, a simple forward difference (for DAP, a backward difference). For the second derivative a symmetric second difference is used. These techniques were checked by hand. They are valid if the differences are smaller than the approximated function in decreasing order and if they change slowly. A comparison of many values of the function and first and second derivatives revealed that the value of the first derivative is usually 1/10 or less of the value of the initial function, while the value of the second derivative averages 1/2 to 1/10 the value of the first derivative.

Similar graphs of the changing values of the numbers A, B, C, D, and f, together with their first and second derivatives, are shown in Figures 4, 5, 6, 7, and 8. For all of the graphs "1" denotes a graph of the Besselian Day Number, "2" denotes a graph of the Day Number's 1st differences, and "3" denotes the graph of the 2nd differences. Note that only the absolute value of the parameters was graphed. The graphs were prepared for the first 30 days of 1976, but are representative of changes throughout the year. The units are arcseconds for A, B, C, and D, but are seconds of time for f. Except for Figure 5 (the number B), the previous discussion holds. For B the first and second derivatives can be equal, and the second derivative can be larger than the first derivative. However, both derivatives are 100 times smaller than B itself.

5.5 Verification of VERNAL

VERNAL is a subroutine taken from reference 8† (page 26). The verification of this subroutine can be found in the same reference on pages 34 and 37. In order to improve this subroutine even further, the initial (1973) value was recomputed by using the equations of reference 2 (page 42). Examination of the second order terms in references 2 and 8 showed that the time in the second order term required a small change. Also, the leap year update was changed from mixed mode arithmetic to integer arithmetic in order to avoid ambiguity. These changes are listed below.

† The factor of -1/2 on page 26 is a misprint and should be eliminated.

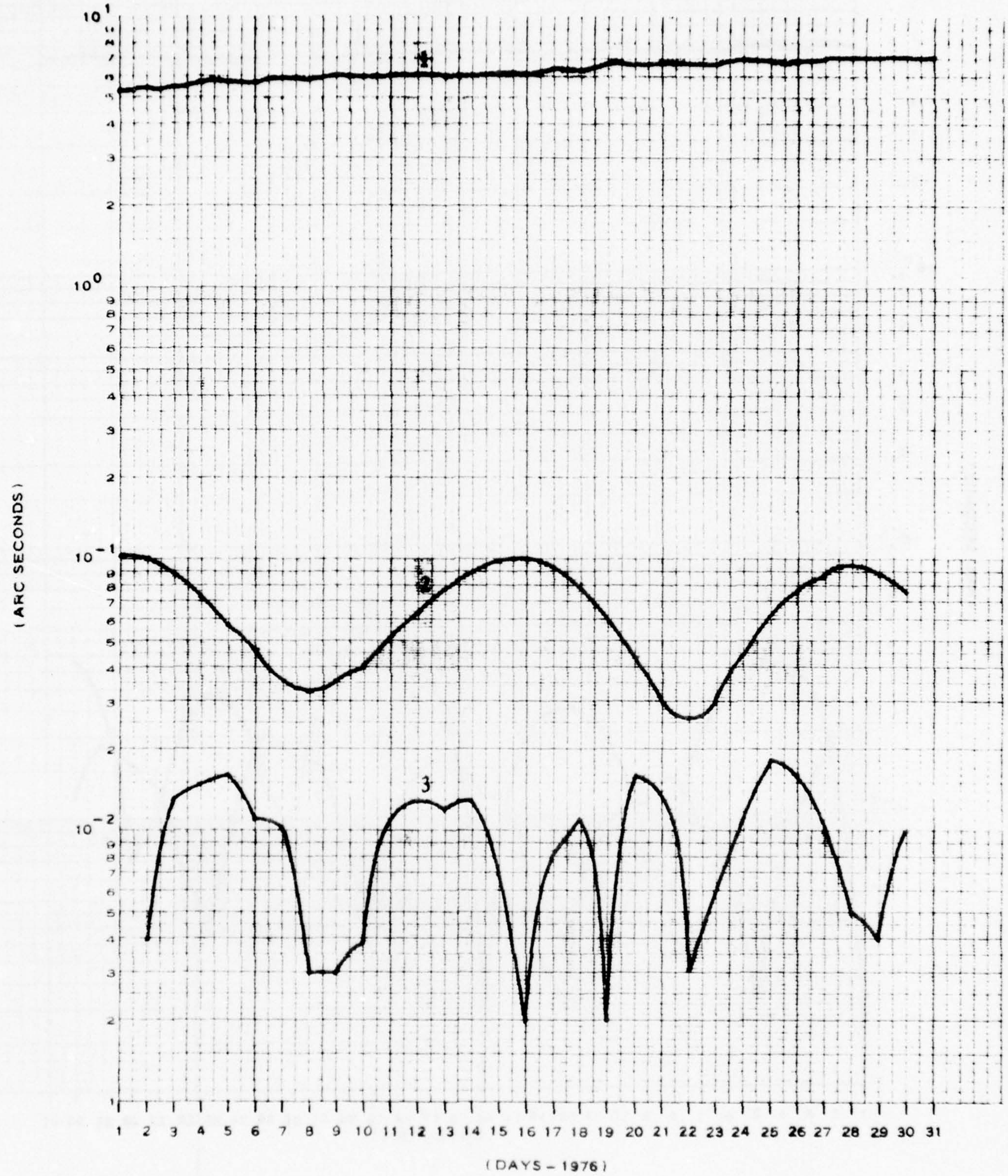


FIGURE 4 BESSELIAN DAY NUMBER A AND 1ST. AND 2ND. DERIVATIVES

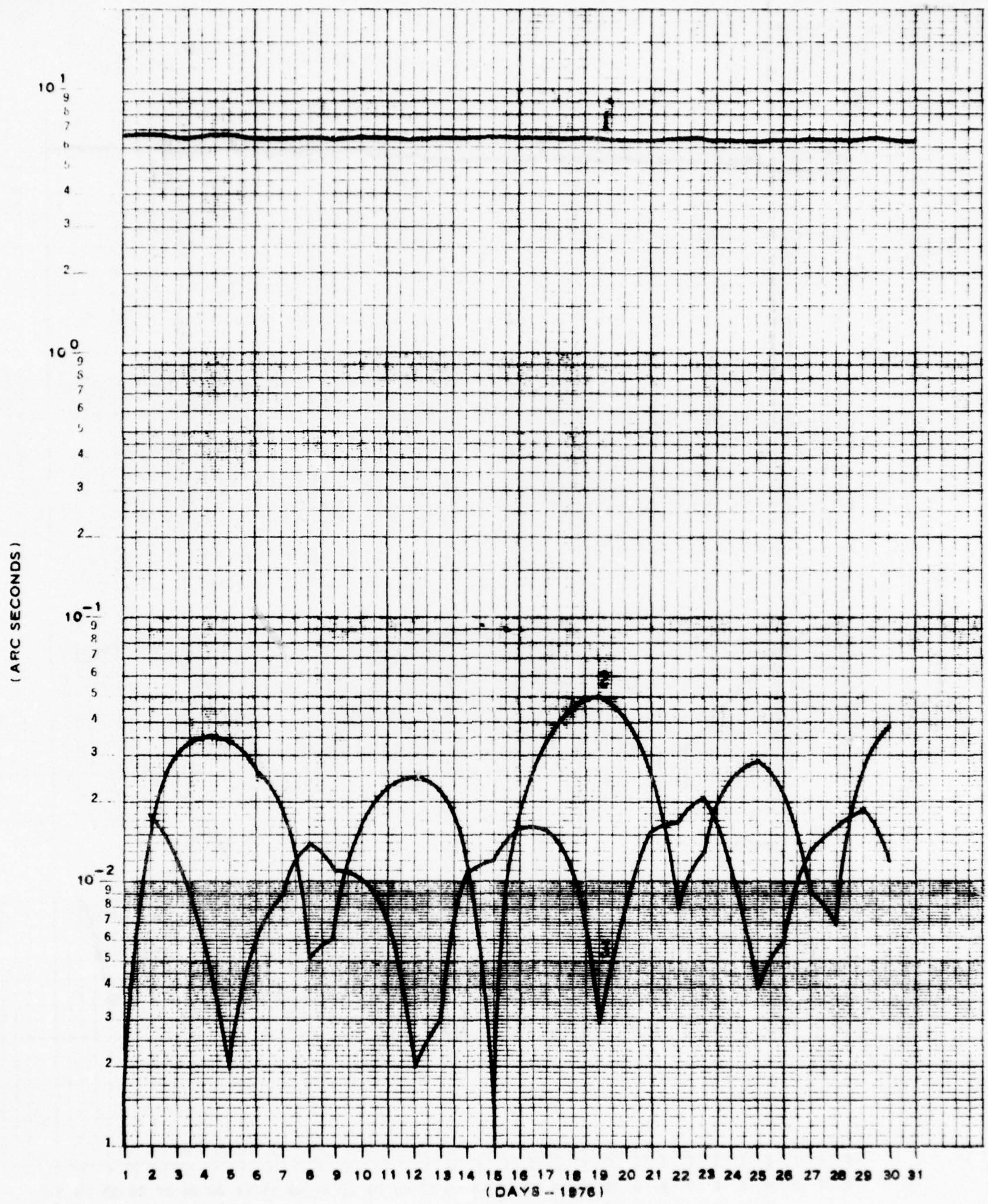


FIGURE 5 BESSELIAN DAY NUMBER B AND 1ST. AND 2ND DERIVATIVES

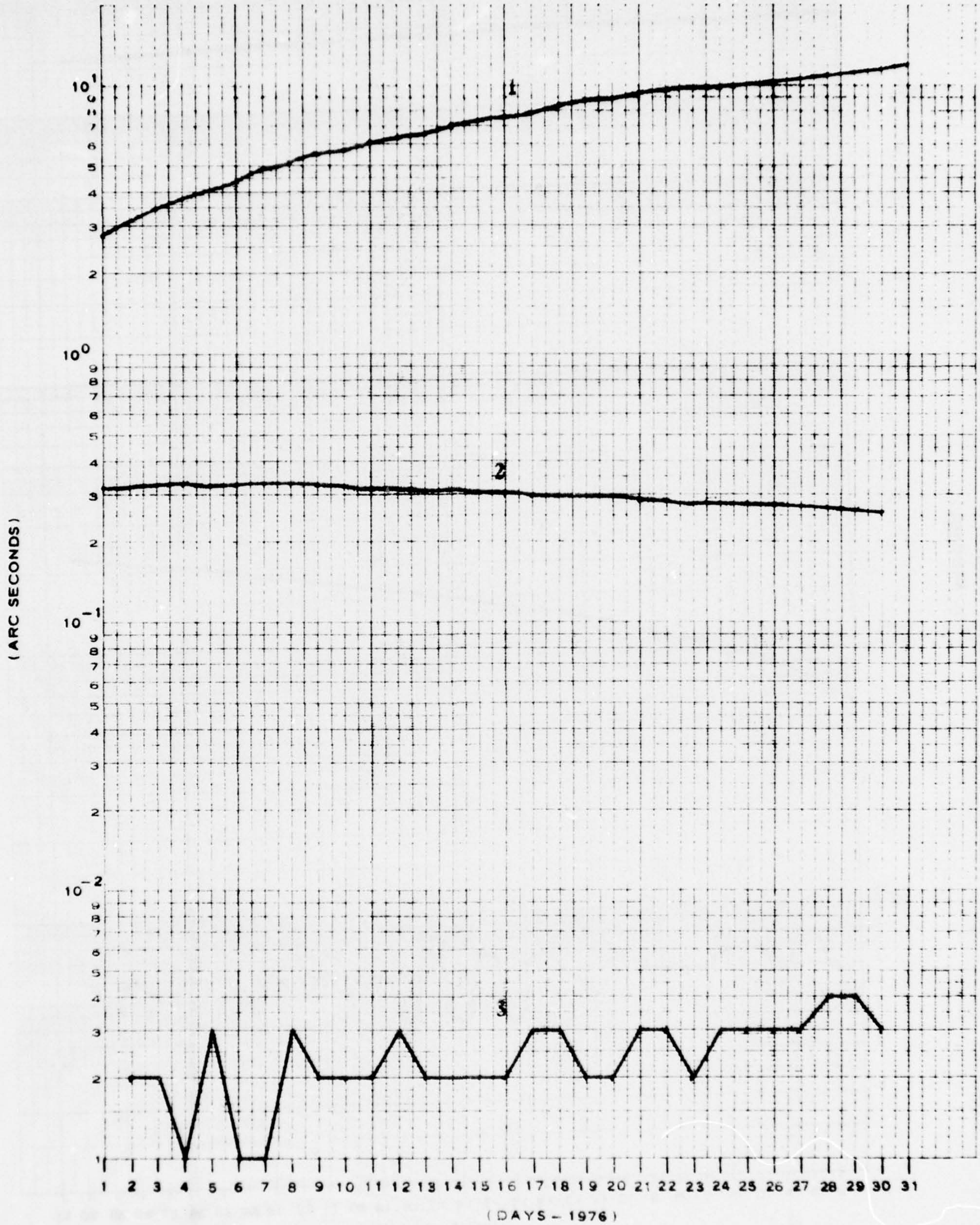


FIGURE 6 BESSELIAN DAY NUMBER C AND 1ST. AND 2ND. DERIVATIVES

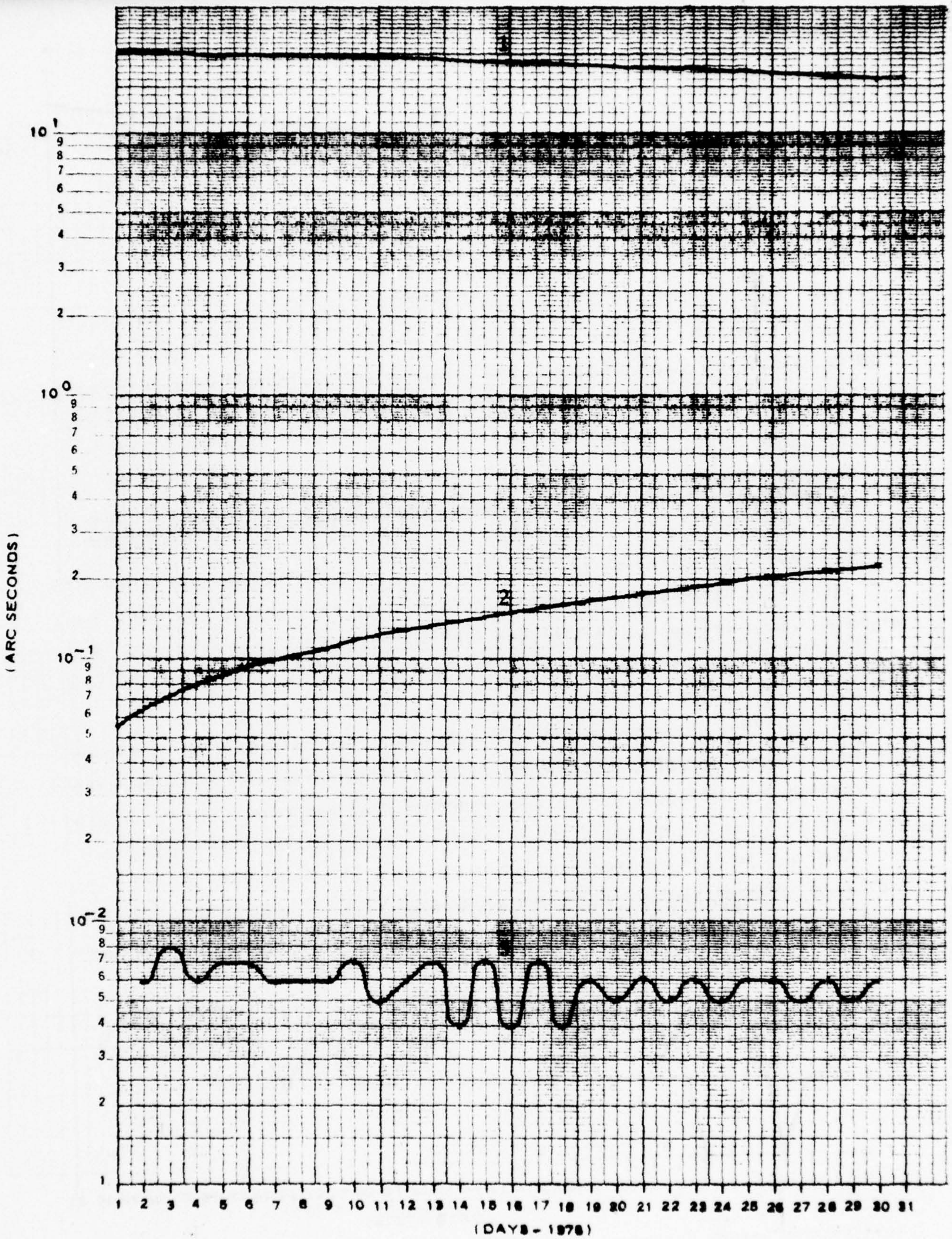


FIGURE 7 BESSELIAN DAY NUMBER D AND 1ST. AND 2ND. DERIVATIVES

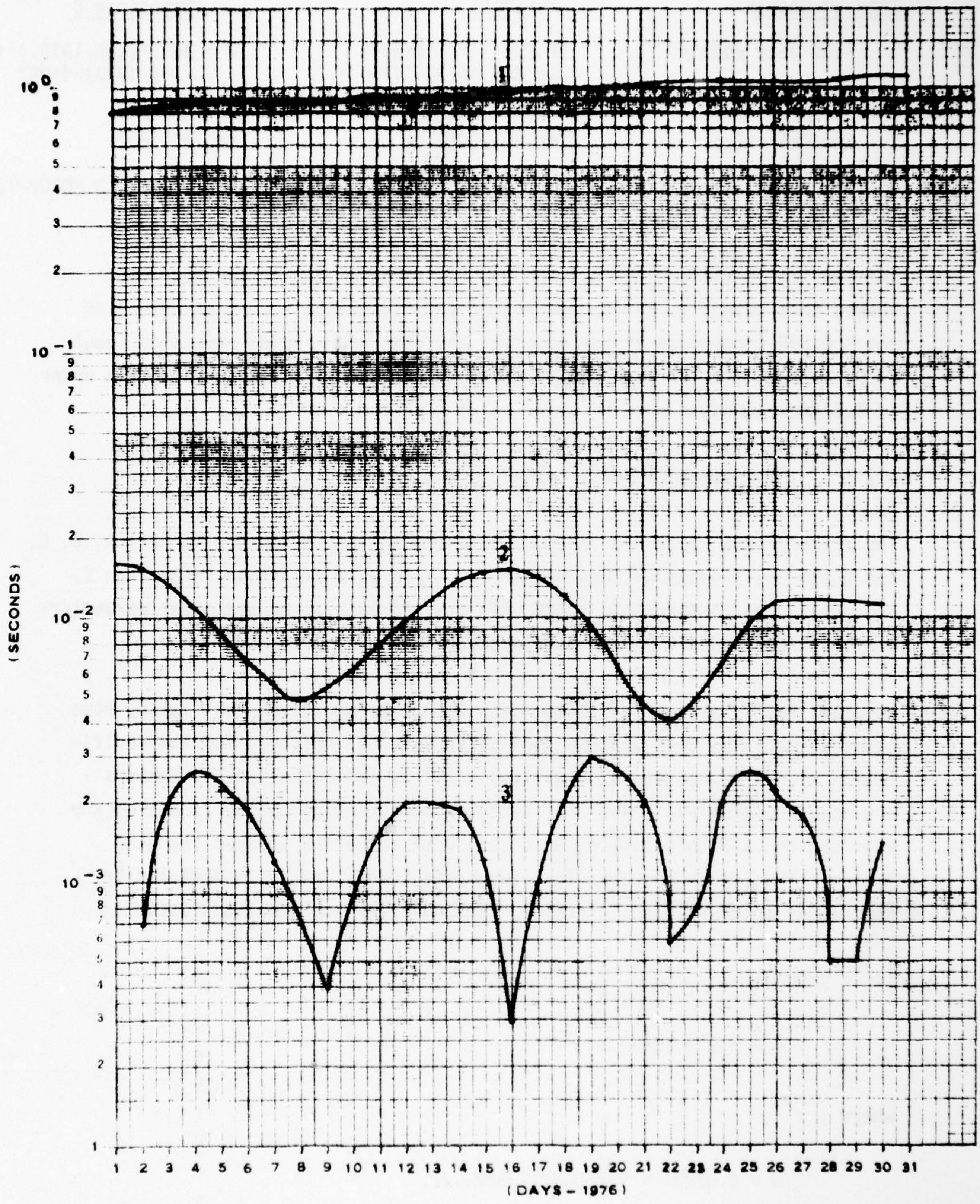


FIGURE 8 INDEPENDENT DAY NUMBER f AND 1ST. AND 2ND. DERIVATIVES

Type of Change	STAR1	Reference 8
1. leap year update	LPYR = (IYR-1973)/4 D = 365.*(IYR-1973) + LPYR+DAY	D = 365.*(IYR-1973.) + (IYR-1973)/4+DAY
2. initial value	260.48673053	260.4867292
3. 2nd order correction	D1 = D + 8400. COR = COR - 2.9015D-13*D1*D1	COR = COR - 2.9015D-13*D*D

Examination of differences between the two forms of VERNAL for year 1976 showed that change 1. had no observable effect, change 3. had an effect of about 0.095 arcseconds at the equator, and change 2. has an effect of about 0.005 arcseconds. For VAFB stations the maximum total effect was on the order of 0.04 arcseconds.

5.6 Discussion of BLOCK DATA

The primary data contained in BLOCK DATA are the Besselian Day Numbers A, B, C, and D and the Independent Day Number f. These were described in Section 2. The other data contained in BLOCK DATA are initial values for other parameters and basic constants. This data can be checked by inspection.

However, the Besselian and Independent Day Numbers must be verified by some other means. The test program used to produce the printout for the verification of CNSTJ provides such a means. All of the Besselian Day Numbers are used to calculate J and J'. Hence, a run printing out every day of the year can be used to verify A, B, C, and D by comparing against the AENA tables. The values for days in between the 10-day values in AENA can be checked by interpolation. Refer to Section 3 for the equations used.

It is known that:

$$f = m\,\tau + \mathrm{EOFE},$$

where

m = 3.07234 + 1.86D-3*T/36524.22,

T = number of days from 1900.0 to the nearest Besselian New Year,

τ = fraction of a tropical year from date to the nearest Besselian New Year, and

EOFE = Equation of Equinoxes.

Hence, EOFE = f - mτ.

A computer program called JJJPP has been designed to check the Besselian Day Numbers and f for a given year in the manner described above. This was accomplished for the BLOCK DATA for 1976. Each year the new BLOCK DATA can be so verified. The program JJJPP is listed in Appendix VIII. This completes the verification of the subroutines.

6.0 VERIFICATION OF MAIN

This program was verified by comparison against certified WTR programs. Of course, this program should be more accurate than previous programs at SAMTEC, so verification does not mean absolute agreement. The programs used for verification were the "mean" to "apparent" transformations on the Sigma 5 computer at the FPQ-14 and the "look angle" output of STASHO. The version of STASHO used was that available on the WANG at PAD. Verbal communication from Bob Baker of FEC/Performance Analysis Department informed the author of an unpublished comparison between the WANG version of STASHO and the 7094 version which showed agreement between these programs within the precision of the program's printout (0.2 arcseconds). The two remaining effects needing verification were the diurnal aberration effect, which was checked against Table 2.6 of reference 4, and the polar motion correction. This last correction was verified by hand calculation.

6.1 Verification of the "Apparent" Position Computation

"Apparent" star positions at 0.0 hour, UTC, for the dates listed in Table 27 were compared against those computed by the FPQ-14 computer. The FPQ-14 data is the 4th Fundamental Catalog of 1950.0 (FK4). For position (Right Ascension, declination) the Mean Star Table of the AENA for the appropriate year (1976 before July 1, 1977 afterward) was used. For both proper motion and parallax the data in reference 9 was used. By use of radial velocities obtained from reference 10 and the data in reference 9, the proper motion parameters were updated via the program STAR2 to the required epoch (1976.0 or 1977.0). The mean star coordinates are listed in Table 26 below.

TABLE 26

Mean Star Coordinates

Name	Epoch	Right Ascension hours, min. sec.	Declination degrees, arcmin., arcsec.	Radial Velocity km/sec	Parallax arcseconds	Proper Motion Rt. Asc. arcseconds/tropical year	Proper Motion Dec. arcseconds/tropical year
Betelguese- O ORI	1976.0	5, 53, 52.3	7, 24, 13	+21.	+.005	+.027	+.007
Castor- A GEM	1976.0	7, 33, 4.2	31, 56, 31.	+4.	+.072	-.165	-.110
Dubhe- A UMA	1976.0	11, 2, 15.5	61, 52, 51.	-9.	+.031	-.119	-.070
Arcturas- A BOO	1976.0	14, 14, 33.9	19, 18, 24.	-5.	+.090	-1.10	-2.00
Rigel- B ORI	1976.0	5, 13, 23.0	-8, 13, 42.	+21.	-.003	+.001	0.0
Procyon- A CMI	1976.0	7, 38, 2.8	5, 17, 15.	-3.	+.288	-.707	-1.03
Alioth- E UMA	1976.0	12, 52, 58.6	56, 5, 23.	-9.	+.008	+.113	-.011
Bellatrix- G ORI	1977.0	5, 23, 53.8	6, 19, 48.	+18.	+.024	-.006	-.014
Alhena- G GEM	1977.0	6, 36, 23.0	16, 25, 12.	-13.	+.031	+.047	-.046
Vega- A LYR	1977.0	18, 36, 9.5	38, 45, 41.	-14.	+.123	+.2000	+.2811
Aldebaran- A TAU	1977.0	4, 34, 35.9	16, 27, 51.	+54.	+.048	+.069	-.190
Wezen- D CMA	1977.0	7, 7, 27.3	-26, 21, 21.	+34.	-.018	-.004	+.003
Antares- A SCO	1977.0	16, 27, 59.6	-26, 22, 56.	-3.	+.019	-.009	-.028

Note that the initial values are accurate to only $\pm.5$ arcsecond in declination and $\pm.05$ second in Right Ascension. This is much lower than the accuracy of the 4th Fundamental Catalogue. Nevertheless, in Table 27 below good agreement is obtained between the two programs. The differences are tabulated in the usual manner; "STAR1" data - "FPQ-14" data. The average error in Table 23 is -.214 arcseconds in declination and +.018 seconds in Right Ascension.

TABLE 27

Comparison of Apparent Positions Obtained from STAR1 Data and from FPQ-14 Sigma 5 Data

Star Name	Date UTC	Right Ascension seconds	Declination arcseconds
Arcturas	1/5/1976	-.006	-.402
Betelguese	"	+.026	-.606
Castor	"	+.036	-.313
Dubhe	"	-.017	+.175
Alioth	5/10/1976	+.007	-.558
Procyon	"	+.069	-.039
Rigel	"	+.030	-.039
Alhena	8/15/1976	+.026	-.486
Bellatrix	"	+.064	+.080
Vega	"	+.010	-.212
Aldebaran	11/19/1976	+.001	-.144
Antares	"	-.006	-.182
Wezen	"	-.005	-.052

6.2 Verification of the Diurnal Aberration Correction

In reference 4, P.50, is a table of diurnal aberration corrections for use when a star is in transit - that is, when the local hour angle is zero or twelve. Comparisons were made with STAR1 by use of special printouts and linear interpolation.

The apparent star declination is 89.1569° (for Polaris). The site latitude is 37.5°. From the table referenced above, the correct values (after linear interpolation) are:

$$\Delta RA = \pm 17.2374, \ \Delta DEC = 0.0, \text{ both in arcseconds.}$$

The values obtained from STAR1 are listed in Table 28.

TABLE 28

Diurnal Correction Data from STAR1

Set #	Local Hour Angle hours	ΔRA arcseconds	ΔDEC arcseconds
1	11.7280	-17.179	+.0180
1	12.2294	-17.191	-.0150
2	23.6136	+17.166	-.0256
2	0.1150	+17.246	+.0071
3	23.5480	+17.131	-.0299
3	0.0494	+17.251	+.0033

In Table 29 the values at local hour angle 12. or 0. (obtained by linear intepolation) are listed, together with their differences from the tabular values (listed in the form "STAR1" - "Table"). The results are within the error resulting from linear interpolation.

TABLE 29

Diurnal Correction Comparison Test

Set #	LHA hours	ΔRA arcseconds	ΔDEC arcseconds	ΔRA-Table arcseconds	ΔDEC-Table arcseconds
1	12.0	-17.1855	+.0001	-.0520	+.00010
2	0.0	+17.228	+.00003	-.0094	+.00003
3	0.0	+17.239	-.00001	+.0016	-.00001

6.3 Verification of "Look Angle" Computation

The data from STAR1 was compared against the WANG version of STASHO as previously mentioned. The input data was:

Mean Right Ascension	= 6 hrs., 29 min., 8.1 sec.
Mean Declination	= 56°, 24', 22.1"
Epoch	= 1976.0
Site Latitude	= +37.4958°
Site Longitude	= +237.4961° E. Long.

All other parameters (except the site encoder size) are zero. For one time period a declination of 36°24'22.1" was used. For all dates after July 1 the mean parameters above were updated to the 1977.0 epoch via STAR2.

The comparison results are listed in Table 30. The units are mils, which were the units used by STASHO. Note that .001 mils ≃ .2 arcseconds. For comparison purposes two STAR1 decks were used. The 'Full' deck is the standard STAR1 deck. The 'Deleted' deck had all processing not common to both STAR1 and STASHO zeroed out. The diurnal aberration correction is the largest of the missing corrections. The differences in Table 30 are listed in the form; "STAR1" - "WANG".

TABLE 30

Comparison of Look Angle Output Between STAR1 and STASHO

Date (UTC) Month/day Hour	"Deleted" Version Azimuth Diff. mils	Elevation Diff. mils	"Full" Version Azimuth Diff. mils	Elevation Diff. mils
1-1/3	+.001	+.001	0.0	0.0
7	0.0	+.002	-.001	+.002
13	-.001	-.001	-.001	-.001
19	+.001	0.0	0.0	+.001
23.5	+.002	+.001	-.001	+.001
1-4/8	-.003	+.001	-.004	+.002
7	+.001	-.001	0.0	-.002
13	+.001	0.0	+.002	+.001
19	+.001	+.002	0.0	+.002
23.5	-.001	+.001	-.002	+.002
1-7/17	+.001	0.0	0.0	-.001
7	+.001	+.001	0.0	+.001
13	+.001	+.001	-.001	0.0
19	-.001	0.0	0.0	+.002
23.5	+.001	0.0	-.002	0.0
1-10/25	+.001	+.001	0.0	+.001
7	+.001	+.001	-.001	+.001
13	0.0	+.001	+.001	+.002
19	0.0	0.0	-.001	+.001
23.5	+.001	+.001	-.002	+.001
Declination Changed to 36°24'22.1", Epoch 1976.0				
1-10/25	-	-	-	-
7	+.001	+.001	0.0	0.0
13	+.002	0.0	-.006	0.0
19	+.001	0.0	0.0	+.001
23.5	-	-	-	-

Note that the maximum error in the "Deleted" Version column is 0.6 arcseconds with an average of 0.12 arcseconds. Since these computations are close to the accuracy of the WANG (considering all computations), STAR1 is verified. The "Full" version has a maximum error 1.2 arcseconds, which is consonant with the lack of correction of the diurnal aberration in the WANG program.

6.4 Termination of Program

In Section 2 the three abnormal terminations available in STAR1 were discussed. These terminations are labeled STOP's. They occur when:

1. The first day of the run is less than or equal to 0.
2. The first day of the run preceeds July 1 while the last day equals or follows July 1.
3. The last day of the run is equal to or greater than January 2 of next year.

Note that the "days" used above are the GMT equivalents of the local start and stop days, rather than the local day numbers themselves.

Since the abnormal STOP's are labeled, the HASP printout will show this. Since the HASP printout is the first page of the run, such conditions can be quickly recognized. Both the HASP and STAR1 printouts for the abnormal cases and the STAR1 printout for the normal case are shown in Figure 9. It is of interest that the year number in the normal printout is preset in BLOCK DATA. Hence, it does not require a special change each year.

6.5 Comment on Refraction Corrections

The output of STAR1 without refraction correction is more than adequate for differencing with real data in order to regress for systematic errors after the real data has been precorrected for refraction errors. If the STAR1 output is to be used for pointing instruments, then the built-in refraction correction is the minimal correction needed. However, this

FIGURE 9
Samples of the Different STAR1 Terminations

```
Case 1                          H A S P   S Y S T E M   L O G

$ 16.26.25 JOB 239 -- STAR1    -- BEGINNING EXEC - INIT 4 - CLASS
 *16.42.00 JOB 239 IHC002I STOP      1
$ 16.42.03 JOB 239 END EXECUTION.

                                     STAR1 PRINTOUT

START DAY IS NEGATIVE OR ZERO FOR THE 1976 VERSION OF STAR1.
PLEASE USE VERSION 1975 AND DAY NUMBERS 365 OR 366 FOR THIS DAY(DAYS).

Case 2                          H A S P   S Y S T E M   L O G

$ 16.41.38 JOB 152 -- STAR1     -- BEGINNING EXEC - INIT 9 - CLASS A
 *17.21.40 JOB 152 IHC002I STOP       2
$ 17.21.46 JOB 152 END EXECUTION.

                                     STAR1 PRINTOUT

          CHANGEOVER TIME
**********************************************************************
BREAK RUN INTO TWO PARTS.  END 1ST PART ON 183.  START 2ND RUN ON 183.
USE STARS REFERENCED TO START OF BESSELIAN YEAR 1976 FOR 1ST RUN.
 "    "     "       "   "   "    "     "  1977  "  2ND  " .
**********************************************************************

Case 3                          H A S P   S Y S T E M   L O G

$ 16.56.39 JOB  87 -- STAR1     -- BEGINNING EXEC - INIT 6 - CLASS A
 *17.10.48 JOB  87 IHC002I STOP       3
$ 17.11.03 JOB  87 END EXECUTION.

                                     STAR1 PRINTOUT

  THE STOP DATE IS NEXT YEAR
  END RUN ON DAY 367.  USE NEW VERSION OF PROGRAM(VERSION 1977) FOR CONTINUATION OF RUN,
  STARTING ON DAY 1 AND ENDING ON DAY 2.

                                     STAR1 PRINTOUT

Normal Termination
   ******************
   END OF PROGRAM
   ******************

THIS IS VERSION 1976, GOOD ONLY FOR YEAR 1976.
CALL YOUR LOCAL PAD CONSULTANT IF YOUR VERSION NEEDS UPDATING
```

program was designed to accommodate more extensive and elaborate refraction subroutines as required by the user. These subroutines are already available at WTR and need not be rediscovered. Local experts on such routines are Mr. Jerry Trimble and Dr. Ruey Han, both of FEC/Performance Analysis Department.

6.6 Conclusions and Recommendations

There is no doubt that this program will produce data to an accuracy of 0.1 arcseconds. Whether or not the data is accurate to 0.01 arcseconds, as was the design criterion, will require checkout against a more accurate program than is currently available at SAMTEC. The references used as background for this program claim an accuracy for the apparent star position in excess of 0.01 arcseconds.

The only remaining problem is that of changing the BLOCK DATA each year (see Appendix V). There are at least two approaches available to further simplify this task. The first approach would be to implement subroutines to obtain the additional astronomic data now missing. If that happens, then the program STAR3 would be used to calculate the Besselian Day Numbers C and D, in addition to A, B, and f. Unfortunately, the technical character of these computations requires no small effort and time to program successfully. The second approach involves obtaining a tape with the required numbers from the Naval Observatory. Then a program would have to be written to read the tape and output the data in useable form.

APPENDIX I

USER INSTRUCTIONS

STARI PROGRAM

Prepared by: Dr George Sinclair Date: 8/2/1977
Dr. George Sinclair
Performance Analysis Department

I. PURPOSE

The purpose of this program is to compute look angles (azimuth and elevation) for tracking a given star from a given site. The output (for a given site and a given star) is azimuth and elevation in the common radar system - Earth rotating, right handed, referenced to the astronomic vertical - and is listed in degrees (and decimal degrees), mils, and octal. Other outputs are time (both local and UTC), plunge azimuth, and plunge elevation. The last two are listed in mils and octal only.

The octal output requires special consideration. This output must be referenced to the number of bits in the site encoders. Hence, if octal output is desired, the bit size of the site encoders must be known. This output is eliminated when a bit size of zero is input.

II Logical Setup of Input Deck

The following is a block diagram of the input deck. Note that the first and last cards appear only once in the deck. Card blocks 2, 3, and 5 must appear in every run. Card block 4 will appear in every run for which polar motion is corrected, but not otherwise.

1 — Number of runs

Run 1

<u>Run Cards</u>

2
- Card for number of sites (I) and number of stars per site J(I) for run 1.
- Integration card for run 1

<u>Station Cards</u>

3
- Station coordinate card for site 1
- Station options card for site 1
- Station coordinate card for site 2
- Station options card for site 2
- ⋮
- Station coordinate card for site I
- Station options card for site I